Proposed Updates to RSM TREs

(Updates to the Replacement Sensor Model Tagged Record Extensions Specification for NITF 2.1)

draft

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Prepared for the NGA

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INTRODUCTION

Three new RSM TREs are documented. They are modifications to three of the current eight TREs, i.e., RSMDCA, RSMAPA, and RSMECA. The new TREs are termed RSMDCX, RSMAPX, and RSMECX, respectively. The last character "X" designates an experimental or prototype version. In the future, the "X" will change to either an "A" or a "B", depending on a current analysis of revision impacts.

If changed to "A", the three new TREs will simply replace their earlier draft counterparts. If changed to "B", both "A" and "B" will be applicable. In particular, an RSM generator and an RSM exploiter should be back-wards compatible to work with either. That is, work with RSMDCA/RSMAPA/RSMECA along with the five other unmodified TREs, and work with RSMDCX/RSMAPX/RSMECX along with the five other unmodified TREs.

The "X" TREs extend the capabilities of the initial TREs. All three "X" TREs reference an extended set of RSM adjustable parameters. The extended set improves RSM adjustability and error propagation performance for some sensors and applications. These sensors typically have a very large number of original adjustable parameters. The extended set of RSM adjustable parameters also includes a "basis" option capability. It allows for easy automatic selection of the appropriate set of RSM adjustable parameters by the RSM generator for an arbitrary sensor/application.

In addition, RSMDCX now supports the generation and exploitation of an arbitrarily large direct error covariance. Also, RSMECX now allows for sets of active RSM adjustable parameters that vary from image to image (no TRE format change required).

1.0 RSM Direct Error Covariance (RSMDCX) TRE

1.1 Overview

The RSM direct error covariance provides a statistical description of image support data error for one or more (multiple) images. It statistically references errors in the values of all active RSM adjustable parameters for these images. The Replacement Sensor Model Direct Error Covariance TRE (RSMDCX) provides blocks of the RSM direct error covariance applicable to the associated image.

When the RSM direct error covariance is provided, the RSM TRE Set for the associated image contains at least one RSMDCX TRE. This TRE contains the covariance block (sometimes termed the auto-covariance block or matrix) for the associated image and may contain cross-covariance blocks between it and other images referenced by the direct error covariance. If more cross-covariance blocks for the associated image are applicable than can be contained in one RSMDCX, one or more additional RSMDCX are included in its RSM TRE Set. These additional RSMDCX do not contain the covariance block. Also, a cross-covariance block is never duplicated within an RSMDCX TRE or across multiple RSMDCX TREs for the associated image. (See Section 1.2 for details on the RSM direct error covariance definition and form, including its constituent covariance and cross-covariance blocks.)

In order for an RSM exploiter to assemble and utilize an entire multi-image direct error covariance, it must access each image's RSM TRE Set and all RSMDCX TREs contained therein.

The RSM TRE generation process insures that each relevant cross-covariance block in the RSM direct error covariance is placed in one of the RSMDCX TREs contained within the collective RSM TRE Sets. In order to minimize support data bandwidth, it is recommended that RSMDCX TREs be generated such that no cross-covariance block (or its matrix transpose) is duplicated across RSM TRE Sets. (Note that if the cross-covariance block for image pair i-j is not supplied and the covariance blocks i-i and j-j are supplied, then the cross-covariance block i-j is assumed identically equal to zero (uncorrelated images) by an RSM exploiter.)

The RSM TRE generation process insures a common value for field TID in all RSMDCX TREs in all corresponding RSM TRE Sets. This positively identifies the RSMDCX TREs as corresponding to the same RSM direct error covariance. The TID value is unique to the particular RSM direct error covariance.

The overall design approach described above allows for the specification of very large direct error covariance while still conforming to the size constraints of the RSMDCX TRE. A very large direct error covariance is associated with a large number of correlated images and/or a large number of active RSM adjustable parameters. It typically corresponds to the a posteriori error covariance resulting from a simultaneous image support data adjustment (triangulation) involving many images.

The row dimension for each cross-covariance block contained in an RSMDCX TRE is common and specified in field NROWCB. It equals the number of active adjustable parameters for the associated image (field IID). In addition, at least one RSMDCX TRE for the associated image specifically identifies these active adjustable parameters. The column dimension for each cross-covariance block contained in an RSMDCX TRE is not necessarily common and is specified in field NCOLCB. It equals the number of active adjustable parameters for the corresponding image. This image is also identified by its original full image ID in field IIDI.

(Note that if an RSMDCX includes the identification of the active adjustable parameters for the associated image, their number is also contained in the field NPAR, i.e., NPAR=NROWCB.)

All cross-covariance blocks statistically reference errors in the values of active RSM adjustable parameters – those corresponding to the row dimension and those corresponding to the column dimension. The actual values of these adjustable parameters are zero unless specified otherwise by an RSMAPX TRE for the appropriate image.

1.2 Direct error covariance form

Assume that the direct error covariance is applicable to n images, identified as images 1 through n for simplicity. The direct error covariance matrix CR has the following form:

$$CR = \begin{bmatrix} CR_{11} & CR_{12} & .. & CR_{in} \\ CR_{21} & CR_{22} & .. & .. \\ .. & .. & .. & .. \\ CR_{n1} & .. & .. & CR_{nn} \end{bmatrix}, \text{ where } CR_{ji} = CR_{ij}^T .$$

In general, CR_{ij} is the cross-covariance between image i and image j (cross-covariance block i-j). It is a $m_i x m_j$ matrix, where m_i is the number of active RSM adjustable parameters for image i and m_j is the number of active RSM adjustable parameters for image j. If R_i corresponds to the vector of (ordered) active RSM adjustable parameters for image i, and εR_i the corresponding errors in its value, the cross-covariance matrix CR_{ij} is formally defined as $CR_{ij} = E\{\varepsilon R_i \varepsilon R_j^T\}$. (All errors are assumed to have a mean value of zero.)

An RSMDCX TRE contained in the RSM TRE Set for associated image k ($1 \le k \le n$) contains up to n CR_{kj} cross-covariance blocks, where the values of j must satisfy $1 \le j \le n$. If $j = k$, this cross-covariance block is also called the (auto) covariance block. The covariance block C_{kk} is symmetric, while all other cross-covariance blocks CR_{kj} are in general rectangular, and even if square are (internally) non-symmetric.

1.3 Adjustable parameter definitions in support of the direct error covariance and error propagation

1.3.1 Overview

As mentioned previously, the active RSM adjustable parameters for the associated image are identified in the RSMDCX TRE. The associated image's cross-covariance blocks are relative to errors in their values. Thus, application of the associated image's cross-covariance blocks requires the complete identification and definition of these adjustable parameters.

In particular, their definition is required in order to compute the partial derivatives of image measurements with respect to the adjustable parameters. These partial derivatives support error propagation, and are utilized in both geopositioning and triangulation solutions. The following sections provide remaining details required for their complete definition. (In general, a cross-covariance block is also relative to errors in the values of adjustable parameters from the other image. The identification and definition of these active RSM adjustable parameters are also required, as discussed below.)

Error propagation associated with the entire direct error covariance requires the identification and definition of each image's active RSM adjustable parameters, supplied in that image's RSMDCX TREs. If we define CR as before, and define $I* = \begin{bmatrix} .. & I^T_{p_i i} & .. \end{bmatrix}^Y$ as a vector of two-dimensional image measurements ($I^T = \begin{bmatrix} r & c \end{bmatrix}$) corresponding to multiple measurements (p_i) made in multiple images $i = 1,..,n$, statistical propagation of support data error to image space for all images is accomplished as follows:

$$CI = (\partial I^* / \partial R^*) CR (\partial I^* / \partial R^*)^T \text{, where } R^* = \begin{bmatrix} R_1^T & .. & R_n^T \end{bmatrix}^T.$$

If $I*$ is a $s \times 1$ vector ($s = \sum_{i=1}^{n} 2 p_i$) and R^* a $t \times 1$ vector ($t = \sum_{i=1}^{n} m_i$, m_i the number of active RSM parameters for image i), CR is the txt direct error covariance, $(\partial I^* / \partial R^*)$ is the $s \times t$ partial derivative matrix, and CI the $s \times s$ error covariance corresponding to errors in the image measurements I^* due to support data errors εR^*. Note that all correlation of errors between all active RSM adjustable parameters and between all images is accounted for in CR, and their corresponding effect on all image measurement errors is statistically specified (including correlations) in CI.

The RSMDX TRE defines and identifies the active RSM adjustable parameters for the associated image as defined in the following sections 1.3.2 - 1.3.4. Note that the associated identification/definition TRE fields are conditional in the RSMDCX TRE and are only included when the field INCAPD is set equal to "Y". If there are multiple RSMDCX TREs for the associated image, INCAPD must equal "Y" for at least one. Use of the field INCAPD allows for the reduction of image support data bandwidth when multiple RSMDCX are required for the associated image.

1.3.2 Definitions and Identification of adjustable parameters

Active RSM adjustable parameters for the associated image are either active RSM "image-space" adjustable parameters or active RSM "ground-space" adjustable parameters, as specified by field APTYP in the RSMDCX TRE.

RSM "image-space" adjustable parameters correspond to adjustable parameters that adjust an image row coordinate value (r) and an image column coordinate value (c) corresponding to an arbitrary ground point location $X = \begin{bmatrix} x & y & z \end{bmatrix}^T$. An individual adjustable parameter either adjusts an image row coordinate value or an image column coordinate value. The adjustments Δr and Δc corresponding to adjustable parameters ap_{rijk} and ap_{cijk} are computed as follows:

$$\Delta r = ap_{rijk} x^i y^j z^k$$

$$\Delta c = ap_{cijk} x^i y^j z^k$$

The adjustable parameters (ap_{rijk} and ap_{cijk}) are uniquely identified by their collective x, y, z powers (exponents) and whether they adjust image row or image column coordinates. The coordinates x, y, and z correspond to normalized ground point coordinates expressed in a Local coordinate system. Normalization is performed by an offset and scale factor for each coordinate. These normalization parameters are in contiguous fields (NSFX-NOFFZ), and allow for an approximate range of (-1,1) for each ground coordinate value. An example of their application for normalization of the y coordinate is as follows:

$$y \to (y - offset_y) / scale_y \, .$$

Because the ground coordinates are normalized, all "image-space" adjustable parameters have units of pixels, as do the corrections $\mathrm{A}r$ and Δc. Normalization of the Local coordinates helps to insure overall stability since the value of $x^i y^j z^k$ that multiplies an adjustable parameter during an image row or column adjustment can become extremely large if coordinates are not normalized for large images and large exponents.

There are two possible choices for the Local coordinate system, either Local Rectangular or Local Non-Rectangular, as specified in field LOCTYP. For Local Non-Rectangular, x, y, and z correspond to the ground point's corresponding image row coordinate, image column coordinate, and geodetic height, respectively. The Local Rectangular coordinate system is defined as a rectangular system that is offset and rotated relative to the WGS-84 coordinate system. It is typically specified as a local tangent plane system centered within the RSM image domain’s footprint at a nominal height above the ellipsoid and rotated to be aligned as follows: the z-axis is aligned with the imaging locus direction (line-of-sight vector for an electro-optical sensor), the x-axis is aligned with the image line (“sweep” or "scan") direction, and the y-axis completes a right-handed rectangular system. (When the Local Rectangular coordinate system is footprint centered, corresponding Local Rectangular coordinate normalization offsets, such as $offset_y$, typically have a value of zero.)

Figure 1 illustrates a typical Local Rectangular coordinate system. Specification of a Local Rectangular coordinate system is unique to the associated image and based on contiguous fields (XUOL-ZUZL) as detailed later in this introduction.

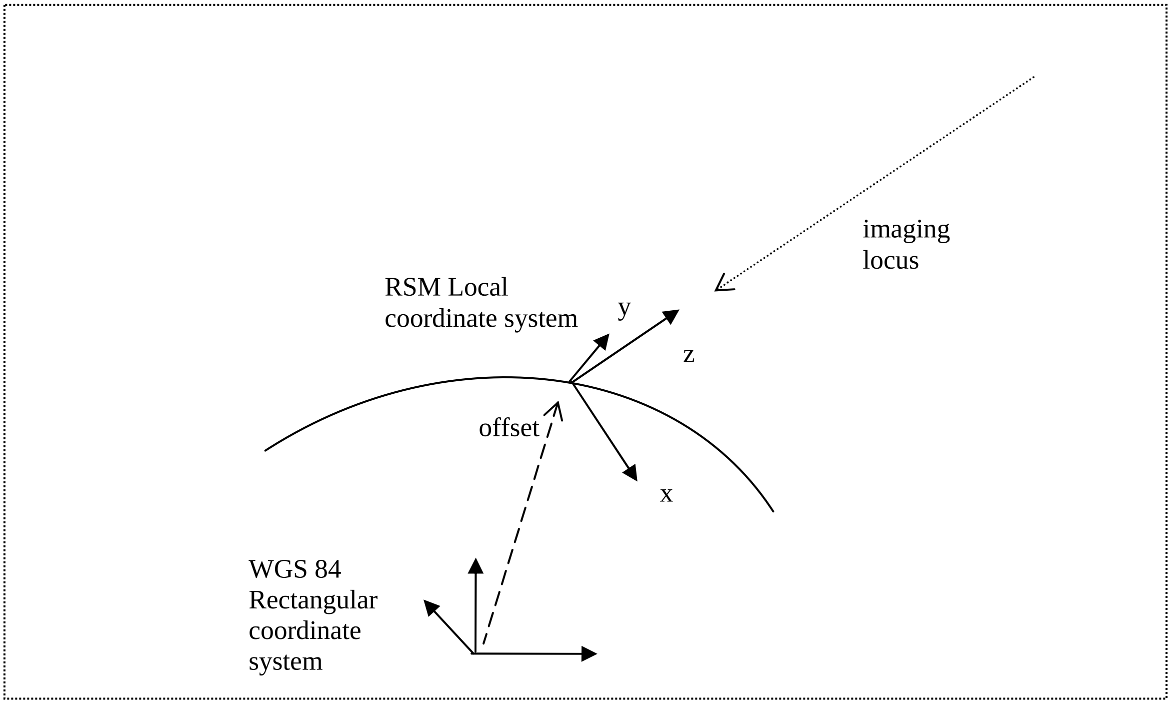

Figure 1: Example of RSM Local Rectangular Coordinate System

Note that the choice of Local Rectangular or Local Non-rectangular is provided for flexibility. The Local Rectangular coordinate system inherits general analytic advantages associated with rectangular (orthonormal) coordinates, and its absolute orientation is insensitive to any abrupt changes in imaging geometry across the imaging time interval. The Local Non-Rectangular coordinate system may provide advantages when very long images are (smoothly) scanned due to significant changes in instantaneous image geometry from one end of the image to the other. The coordinate system is continuously in alignment with these changes.

RSM "ground-space" adjustable parameters reference normalized Local Rectangular coordinates only. The coordinate system is typically specified as a local tangent plane system centered within the RSM image domain’s footprint at a nominal height above the ellipsoid (z-axis vertical). An individual "ground-space" adjustable parameter is either a parameter associated with a "seven parameter" adjustment or a "rate" adjustment. The seven parameter adjustment is defined as follows, where the symbols $\{\delta x \quad \delta y \quad \delta z \quad \delta\alpha \quad \delta\beta \quad \delta\kappa \quad \delta s\}$ correspond to the adjustable parameters:

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + \begin{bmatrix} \delta s & \delta\kappa & -\delta\beta \\ -\delta\kappa & \delta s & \delta\alpha \\ \delta\beta & -\delta\alpha & \delta s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The vector on the left side of the above equation corresponds to adjustments in the un-normalized coordinates of the ground point expressed in Local Rectangular coordinates with units of meters. The vector on the far right side of the equation corresponds to normalized coordinates of the ground point expressed in Local Rectangular coordinates. Because these coordinates are unit-less, the adjustable parameters all have units of meters, as do the corrections $\Delta x, \Delta y, \Delta z$.

(If Local coordinate scale factors ($scale_x, scale_y, scale_z$) are set equal in value by the TRE generation process, Local coordinate values no longer necessarily range from -1.0 to 1.0. However, the above seven parameter adjustment is now equivalent to the standard photogrammetric seven parameter (small angle) transformation. It is recommended that the scale factors be set equal to a common value in a manner that yields ranges for the three local coordinates as close as possible to the interval -1.0 to 1.0.)

There are nine possible ground-space adjustable parameters corresponding to rate adjustments and denoted by the symbols $\{ap_{xx}, ap_{xy}, .., ap_{zz}\}$. They adjust the un-normalized coordinates of the ground point in Local Rectangular coordinates specifically as follows:

$\Delta x = ap_{xx}x$, $\Delta x = ap_{xy}y$, $\Delta x = ap_{xz}z$,

$\Delta y = ap_{yx} x$, $\Delta y = ap_{yy} y$, $\Delta y = ap_{yz} z$,

$\Delta z = ap_{zx} x$, $\Delta z = ap_{zy} y$, $\Delta z = ap_{zz} z$.

Again, these adjustable parameters and the corrections have units of meters.

Each of the 16 possible ground-space adjustable parameters is identified by a unique four character identifier detailed in the TREs specified format (Table 1).

Note that application of RSM adjustable parameters, whether "image-space" or "ground-space" adjustable parameters, first requires converting the corresponding ground point from representation in the RSM primary ground coordinate system to the appropriate Local system. And for the case of "ground-space" adjustable parameters, the adjusted ground point must also be converted back to the RSM primary coordinate system.

Figure 2 presents the RSM adjustable ground-to-image function $h(X,R)$, where X corresponds to the un-normalized three dimensional ground point in the RSM primary ground coordinate system. The functions $I\_adj(X,R)$ and $X\_adj(X,R)$ apply the previously documented adjustment equations for active "image-space" and "ground-space" adjustable parameters, respectively. (The functions also internally convert X from the primary system to the (normalized) Local system.)

$\Delta I = [\Delta r \quad \Delta c]^T$ denotes the summed effects at ground point location X of all active RSM "image-space" adjustable parameters. For example, if the active "image-space" adjustable parameters correspond to (combined) powers in x and y less than or equal to one: $\Delta r = ap_{r000} + ap_{r100} \cdot x + ap_{r010} \cdot y$, and $\Delta c = ap_{c000} + ap_{c100} \cdot x + ap_{c010} \cdot y$.

$\Delta X = [\Delta x \quad \Delta y \quad \Delta z]^T$ denotes the summed effects at ground point location X of all active RSM "ground-space" adjustable parameters.

The vector R represents the active RSM adjustable parameters in the order that they are specified in this populated TRE, e.g., vector element two corresponds to the second active adjustable parameter identified in the populated TRE (see Table 1).

Note that during RSM TRE generation, selection and subsequent specification of active RSM adjustable parameters is independent of selection and subsequent specification of the RSM ground-to-image function (polynomial and/or interpolated grid). In addition, the RSM ground-to-image function $g(X)$ is actually performed with respect to normalized coordinates. The RSM ground-to-image function handles all required normalization and un-normalization, as described in the RSMPCA TRE and the RSMGGA TRE.

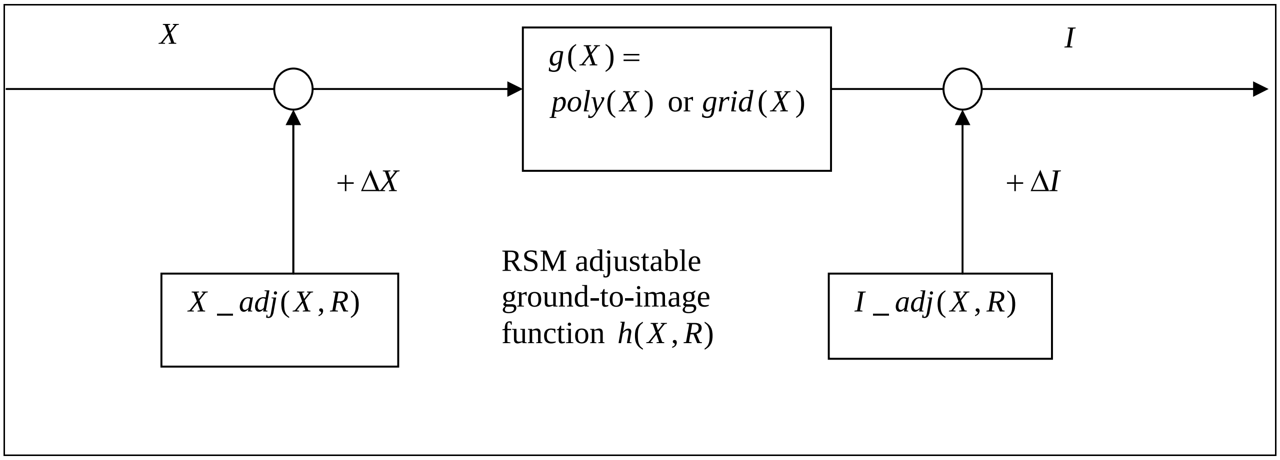

Figure 2: RSM adjustable ground-to-image function

The total number of active "image-space" adjustable parameters is specified in field (NISAP). Individual active adjustable parameters are identified in contiguous fields (XPWRR-ZPWRR) and contiguous fields (XPWRC-ZPWRC). The index of an active adjustable parameter into a cross-covariance block for the associated image corresponds to the order it is identified in the populated TRE.

The total number of active "ground-space" adjustable parameters is specified in field (NGSAP). Individual active adjustable parameters are identified in field (GSAPID). The index of an active parameter into a cross-covariance block for the associated image corresponds to the order it is identified in the populated TRE.

As mentioned earlier, active RSM adjustable parameters require definition and identification in order to support error propagation. In particular, to project the (RSM support data) image error covariance to image space via the partial derivatives of image coordinates with respect to the active adjustable parameters. These derivatives are computed for the various active adjustable parameters by taking the appropriate derivatives of the previous equations.

For example, the partial derivative of the image column coordinate with respect to an "image-space" adjustable parameter is computed as follows:

$$\partial c / \partial ap_{cijk} = \partial \Delta c / \partial ap_{cijk} = \partial (ap_{cijk} x^i y^j z^k) / \partial ap_{cijk} = x^i y^j z^k$$

(Recall that in the above the ground point's location (x, y, z) is represented as normalized coordinates in the Local system.)

Partial derivatives of image coordinates with respect to "ground-space" adjustable parameters are more involved because they adjust image coordinates indirectly. For example, the partial derivative of the image row coordinate with respect to the "ground-space" adjustable parameter ap_{xy} is computed as follows:

$$\partial r / \partial ap_{xy} = (\partial r / \partial X)(\partial X / \partial ap_{xy})$$

In this equation, X represents the ground point's location in the primary coordinate system, and $\partial r / \partial X$ (1×3) is readily computed from the RSM ground-to-image function for the row coordinate, i.e., $\partial r / \partial X = \partial g_r(X) / \partial X$.

The (3×1) $\partial X / \partial ap_{xy} = (\partial X / \partial X_L)(\partial X_L / \partial ap_{xy})$, where X_L represents the ground point location in the (un-normalized) Local Rectangular coordinate system. $\partial X / \partial X_L$ $(3x3)$ is readily computed from the Local Rectangular coordinate system and the primary coordinate system defining parameters. Also, from the definition of ap_{xy}, the (3×1) $\partial X_L / \partial ap_{xy} = \begin{bmatrix} scale_x \cdot y & 0 & 0 \end{bmatrix}^T$, where y is the second component of the ground point's location in normalized Local Rectangular coordinates.

1.3.3 Basis Option

When this option is invoked (APBASE=Y), the set of RSM adjustable parameters specified in this TRE (as described previously) become a "basis" set of adjustable parameters. Symbolically, they are contained in the vector R, now assumed to have n elements. Another set of RSM adjustable parameters is defined as a linear combination of the elements of R. Symbolically, this new set is contained in the vector R', where $R' = AR$, and the matrix A is $m \times n$, $m \leq n$, with the rank of A equal to m. The vector R' has m elements (or more specifically, m_k elements for associated image k). The vector R' contains the (new) set of active adjustable parameters.

The image error covariance (and cross-covariance) contained in this TRE is now with respect to R'. The field NPAR now corresponds to the number of elements (m) in R'. The field NBASIS corresponds to the number of elements (n) in R.

Typically, the image error covariance and A are determined during generation of this TRE from an initial error covariance with respect to the basis set R using principal component analysis. R can also be written as a linear combination of R' based on the pseudo-inverse of A, designated as $A^{\#}$ (or AI). Thus, $R = A^{\#} R'$, where $A^{\#} = A^T (AA^T)^{-1} = A^T$. The equality $A^{\#} = A^T$ follows because the rows of A are unit eigenvectors and members of an orthonormal basis of vectors.

Use of principal component analysis may allow for easier automatic selection of appropriate active RSM adjustable parameters during TRE generation with corresponding positive definite error covariance and a high fidelity relative to the associated image's original sensor model's adjustable parameters and support data error covariance.

For example, the entire collection (RSM Choice Set) of possible RSM adjustable parameters contains over 200 candidates, most corresponding to possible high-order "image-space" adjustable parameters that reference combined powers of Local x, y, and z coordinates. A typical basis set (R) contains a much smaller subset, but still contains a "generous" number of RSM adjustable parameters, such as all "image-space" adjustable parameters that reference combined powers of Local x and y coordinates less than or equal to a (combined) maximum. In addition, if the Local x coordinate is aligned with the image line direction (time) for a pushbroom or "scanning" sensor, the maximum allowed power of the y coordinate may be further restricted less than a maximum allowed power for the x coordinate.

This generous set insures high fidelity of the RSM image error covariance with respect to the corresponding original sensor model image error covariance (projected to image space). However, the RSM image error covariance is either non-positive definite (non-invertible) or has an extremely large condition number, hence, nearly non-positive definite. Principal component analysis of the RSM image error covariance is then used to determine a smaller set (R') of active adjustable parameters as a linear combination of the basis set. This new set still maintains adequate fidelity, but now corresponds to a positive-definite error covariance with reasonable condition number. As an example of the number of adjustable parameters involved with this option, a space-born push-broom sensor may have a basis set (R) with 20 adjustable parameters and a final set (R') with 12 active adjustable parameters.

When the basis option is on (APBASE=Y), the RSMDCX TRE contains the identification of the elements of the basis (R), the matrix A that maps the basis (R) to the set of active adjustable parameters (R'), and the image error covariance (and cross-covariance) with respect to R' for the associated image. Corresponding exploiter functionality is invoked by specification of the appropriate partial derivatives of image coordinates with respect to R', and the update of the RSM ground-to-image function from values of R' solved for during an RSM adjustment (triangulation):

(1) $\partial I / \partial R' = (\partial I / \partial R) A^T$, and

(2) values of R' map to values of R via $R = A^T R'$, where the subsequent R affects the ground-to-image function.

The first of these equations supports the statistical propagation of RSM support data errors to image space, i.e., the projection of the associated image's error covariance (and cross-covariance) contained in this TRE to image space. (The computation of the partial derivatives $\partial I / \partial R$ was documented previously). The second equation maps adjustments contained in R' to R for subsequent application in the RSM adjustable ground-to-image function for the associated image.

1.3.4 Local Rectangular coordinate system details

The following defines the Local Rectangular coordinate system relative to the WGS 84 Rectangular coordinate system. The contiguous fields XUOL through ZUOL specify the origin (offset) of the Local coordinate system relative to the WGS 84 Rectangular coordinate system, and the contiguous fields XUXL through ZUZL elements of the rotation matrix. These fields are provided in this TRE.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{LOCAL}} = \begin{bmatrix} XUXL & YUXL & ZUXL \\ XUYL & YUYL & ZUYL \\ XUZL & YUZL & ZUZL \end{bmatrix} \cdot \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{WGS}-84} - \begin{bmatrix} XUOL \\ YUOL \\ ZUOL \end{bmatrix} \right).$$

Note that the definition of the Local Rectangular coordinate system is also redundantly supplied in other TREs for the associated image. Also, in order to convert a ground point X represented in the RSM primary ground coordinate system (e.g., Geodetic) to the Local Rectangular coordinate system, it must first be converted from the RSM primary system to the WGS 84 Rectangular coordinate system.

1.4 Cross-Covariance Matrix element ordering and size

As mentioned previously, cross-covariance blocks (matrices) corresponding to the associated image are provided in this TRE. In particular, the values of the elements of matrices C_{kj} are supplied, where the index k corresponds to the associated image, and the index j corresponds to an image correlated with the associated image (possibly itself), where $1 \leq j \leq n$. There are a total of up to n image cross-covariance blocks. The dimension of a matrix C_{kj} is $m_k \times m_j$, where m_k is the number or active adjustable parameters for the associated image and m_j is the number of active adjustable parameters for image j. (Note that in order to simplify the TRE format, the symmetry of C_{kk} is not exploited since all the other C_{kj} are not necessarily (internally) symmetric.)

Field IID provides the identification of the associated image and field NROWCB the number of its active adjustable parameters (m_k). (The value of field NROWCB also equals the value of field NPAR, when the latter is present). Field NIMGE provides the total number of images involved, i.e., the total number of cross-covariance blocks contained in this TRE. Fields IIDI and NCOLCB provide the identification of each image and its corresponding number of active RSM adjustable parameters (m_j). The elements of a matrix C_{kj} are contained in field CRSCOV and are stored in row major order.

The size of an RSMDCX TRE can not exceed 99,996 bytes. Therefore the number of cross-covariance blocks it may contain is limited, i.e., NROWCB*NCOLCB*(21 bytes/element), summed over all (NIMGE) images, must be less than approximately 99k bytes. Assuming 6 active RSM adjustable parameters per image, approximately 120 correlated images (cross-covariance blocks) can be accommodated in an individual RSMDCX TRE. Assuming 18 active RSM adjustable parameters per image, approximately 13 correlated images can be accommodated in an individual RSMDCX TRE.

For the 18 adjustable parameter case, let us further assume that there are a total of 15 images associated with the entire direct error covariance. Therefore, in order for an RSM exploiter to assemble the entire direct error covariance, it would require receipt of 15 RSM TRE Sets, one for each image, with each RSM TRE Set containing either one or two RSMDCX TREs.

For example: (1) the RSM TRE Set for image 1 contains two RSMDCX that collectively contain image i-j cross-covariance blocks 1-1, 1-2, ..,1-15, (2) the RSM TRE Set for image 2 contains two RSMDCX that collectively contain cross-covariance blocks 2-2, 2-3, ..,2-15, (3) the RSM TRE Set for image 3 contains one RSMDCX that contains cross-covariance blocks 3-3, 3-4, ..,3-15, (4) ... (14) the RSM TRE Set for image 14 contains one RSMDCX that contains cross-covariance blocks 14-14, 14-15, and (15) the RSM TRE Set for image 15 contains one RSMDCX that contains cross-covariance block 15-15. (Because i equals j, this block is actually a covariance (or auto-covariance) block.)

Another example that requires more RSM TRE bandwidth: (1) the RSM TRE Set for image 1 contains two RSMDCX that collectively contain image i-j cross-covariance blocks 1-1, 1-2, ..,1-15, (2) the RSM TRE Set for image 2 contains two RSMDCX that collectively contain cross-covariance blocks 2-1, 2-2, 2-3, ..,2-15, (3) ... (15) the RSM TRE Set for image 15 contains two RSMDCX that collectively contains cross-covariance blocks 15-1, 15-2, ..,15-15. Note that within the individual RSMDCX TREs contained in an image's RSM TRE Set, the cross-covariance blocks may be stored in any order. For example, the first RSMDCX for image 2 may contain (in order) cross-covariance blocks 2-2, 2-1, 2-3, .. ,2-14, and the second RSMDCX, cross-covariance block 2-15.

(The above sizing estimates assume that the definition and identification of the active RSM adjustable parameters requires little RSMDCX TRE bandwidth, as is typical, i.e., the "basis option" is not applicable or there are relatively few associated adjustable parameters , or that the definition and identification of the adjustable parameters are included (INCAPD=Y) in only one of the multiple RSMDCX TREs for the associated image. In addition, the above sizing estimates and restrictions were per TRE. There is also an approximate 1 Gbyte restriction for the sum of all TREs in an RSM TRE Set. This restriction corresponds to the allowed size of the NITF Data Extension Segment. This added restriction is not a factor since approximately 10000 RSMDCX, each approximately 100000 bytes in size, are allowed.)

1.5 Summary

In summary, the RSM direct error covariance can support a wide variety of exploitation activities and is contained in (multiple) RSMDCX TREs. It can statistically represent unadjusted or adjusted RSM image support data errors. It can support either subsequent geopositioning or RSM triangulation. For the former, it provides the foundation for error propagation, i.e., the propagation of the support data error (covariance) to the target position error (covariance). For the latter, it can provide either the a priori error covariance of the RSM adjustable parameters prior to their solution (adjustment), or their a posteriori error covariance after solution. Finally, use of the RSMDCX TRE allows for the representation of an arbitrarily large direct error covariance.

1.6 RSMDCX format

Table1 specifies the detailed format for the Replacement Sensor Model Direct Error Covariance (RSMDCX) TRE.

RSMDCX – Replacement Sensor Model Direct Error Covariance						
Field	**Name/Description**	**Size**	**Form**	**Units**	**Estimated Value Range**	**Type**
TAG Information						
CETAG	Unique Extension Type Identifier Unique TRE identifier.	6	BCS-A	N/A	RSMDCX	R
CEL	Length of User-Defined Data Length in bytes of data contained in subsequent fields. (TREs length is 11 plus the value given in the CEL field)	5	BCS-N	bytes	00269 to 99985	R
Image Information						
IID	Image Identifier. This field contains a character string that uniquely identifies the original full image that corresponds to the associated image. This is not to be confused with the identification of an image derived by filtering, chipping, re-sampling, or other such image to image transformations. The image identifier is left justified with trailing spaces.	80	BCS-A	N/A	N/A All spaces if unavailable	<R>
EDITION	RSM Image Support Data Edition. This field contains a character string that uniquely identifies the RSM support data for the associated original full image. It is to consist of an identifier of up to 20 characters for the processor that generated the RSM support data, to which is appended up to 20 characters that are unique to that processor.	40	BCS-A	N/A	N/A	R

TID	Triangulation ID. This field contains an identifier that is unique to the most recent process after RSM support data generation that led to the adjustments and/or error covariance in this RSM support data edition.	40	BCS-A	N/A	N/A	R
Cross-covariance block information						
NROWCB	Number of rows per block. This field contains the number of rows in each cross-covariance block contained in this TRE. NROWCB is equal to the number of active adjustable parameters for the associated images, i.e., NROWCB=NPAR.	2	BCS-N	N/A	01 to 36	R
Number of images referenced by cross-covariance blocks (column dimension)						
NIMGE	Number of Images. This field contains the number of images corresponding to the cross-covariance blocks contained in this TRE. Each of these images corresponds to the column dimension of a cross-covariance block. NIMGE equals the number of cross-covariance blocks contained in this TRE.	3	BCS-N	N/A	001 to 999	R
…Begin for each image (NIMGE entries)						
IIDI	Image Identifier. This field contains the original full image identification corresponding to an image associated with the columns of a cross-covariance block contained in this TRE. Identifications are listed in the order their corresponding cross-covariance block are stored (see field CRSCOV)	80	BCS-A	N/A	N/A	R
NCOLCB	Number of columns per block. This field contains the number of columns in the cross-covariance block associated with image IIDI. NCOLCB is equal the the number of active adjustable parameters for image IIDI. If IIDI=IID, the cross-covariance block is also a covariance (or auto-covariance) block and NCOLCB=NROWCB.	2	BCS-N	N/A	01 to 36	R
…End for each image						
INCAPD	Include Adjustable Parameter Definitions Flag. This field specifies whether the RSM adjustable parameters for the associated image are identified and defined in the following fields. At least one populated RSMDCX for the associated image must have INCAPD=Y.	1	BCS-A	N/A	Y or N	R
...if (INCAPD=Y)						
RSM Adjustable Parameter Identification for the associated image						

NPAR	Number of Active RSM Adjustable Parameters. This field contains the total number of active RSM adjustable parameters for the associated image. The value of this field is the row dimension of any cross-covariance (block) for the associated image that is contained in this TRE. NPAR's maximum value of 36 constrains the covariance block to be of reasonable size. (If the "basis" option is off (APBASE=N), NPAR=NISAP if APTYP=I, and NPAR=NGSAP if APTYP=G. If the basis option is on (APBASE=Y), NPAR corresponds to the number of (new) active adjustable parameters and the number of rows in the matrix A, as described for field APBASE.)	2	BCS-N	N/A	01 to 36	R
APTYP	Adjustable Parameter Type. This field identifies whether RSM adjustable parameters are "image-space" (APTYP=I) or "ground-space" (APTYP=G) adjustable parameters.	1	BCS-A	N/A	I or G	C
LOCTYP	Local Coordinate System Identifier. This field identifies whether the Local coordinate system references rectangular ground coordinates (LOCTYP=R) or non-rectangular (image row/image column/geodetic height) coordinates (LOCTYP=N). If RSM adjustable parameters are specified as "ground-space" (APTYP=G), the only valid value is LOCTYP=R.	1	BCS-A	N/A	R or N	C
Normalization Factors for the Local System						
NSFX	Normalization Scale Factor for X. This field contains the normalization scale factor for the x component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NSFY	Normalization Scale Factor for Y. This field contains the normalization scale factor for the y component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NSFZ	Normalization Scale Factor for Z. This field contains the normalization scale factor for the z component of the Local coordinate system	21	BCS-A	meters	±9.99999999999999E±99	C
NOFFX	Normalization Offset for X. This field contains the normalization offset for the x component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NOFFY	Normalization Offset for Y. This field contains the normalization offset for the y component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NOFFZ	Normalization Offset for Z. This field contains the normalization offset for the z component of the Local coordinate system.	21	BCS-A	meters	±9.99999999999999E±99	C

Local Rectangular Ground Coordinates Detailed Definition for Associated image						
...if (LOCTYP=R)						
XUOL	Local Coordinate Origin (XUOR). This field provides the WGS 84 X coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E±99	C
YUOL	Local Coordinate Origin (YUOL). This field provides the WGS 84 Y coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E±99	C
ZUOL	Local Coordinate Origin (ZUOL). This field provides the WGS 84 Z coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E±99	C
XUXL	Local Coordinate Unit Vector (XUXL). This field provides the WGS 84 X component of the unit vector defining the X-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
XUYL	Local Coordinate Unit Vector (XUYL). This field provides the WGS 84 X component of the unit vector defining the Y-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
XUZL	Local Coordinate Unit Vector (XUZL). This field provides the WGS 84 X component of the unit vector defining the Z-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
YUXL	Local Coordinate Unit Vector (YUXL). This field provides the WGS 84 Y component of the unit vector defining the X-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
YUYL	Local Coordinate Unit Vector (YUYL). This field provides the WGS 84 Y component of the unit vector defining the Y-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
YUZL	Local Coordinate Unit Vector (YUZL). This field provides the WGS 84 Y component of the unit vector defining the Z-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C

ZUXL	Local Coordinate Unit Vector (ZUXL). This field provides the WGS 84 Z component of the unit vector defining the X-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
ZUYL	Local Coordinate Unit Vector (ZUYL). This field provides the WGS 84 Z component of the unit vector defining the Y-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
ZUZL	Local Coordinate Unit Vector (ZUZL). This field provides the WGS 84 Z component of the unit vector defining the Z-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E±99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
...end if (LOCTYP=R)						
RSM Adjustable Parameters Basis Option						

APBASE	Basis Option. This field indicates whether the RSM adjustable parameters "basis" option is on (APBASE=Y). If this option is off (APBASE=N), the RSM adjustable parameters specified in the following fields are the active RSM adjustable parameters. The order (component number) of an active RSM adjustable parameter is the order in which it is specified. If this option is on, the RSM adjustable parameters specified in the following fields are the basis set of RSM adjustable parameters. The order (component number) of a basis RSM adjustable parameter is the order in which it is specified. If this option is on, the active RSM adjustable parameters are a linear combination of the basis set of RSM adjustable parameters. The matrix A (field AEL) maps the basis set to the active set of RSM adjustable parameters. In addition, the pseudo-inverse of the matrix A is equal to the matrix A transpose. It maps the active set to the basis set of RSM adjustable parameters. The matrix A is mxn, where $m \le n$ and the rank of A equals m. The number of adjustable parameters (n) in the basis set is specified in field NBASIS. The number of active adjustable parameters (m) is specified in the field NPAR. The RSM image error covariance is always with respect to the active RSM adjustable parameters. For example, the second active RSM adjustable parameter corresponds to row 2 and column 2 of the image (auto) covariance, and corresponds to row 2 and column k of the cross-covariance of the associated image with image k.	1	BCS-A	NA	Y or N	C
"Image-Space" Adjustable Parameters						
,,,if (APTYP = I)						

NISAP	Number of Image-Space Adjustable Parameters. This field contains the total number of image-space adjustable parameters. If the basis option is off (APBASE=N), specified image-space adjustable parameters are the active RSM adjustable parameters. The total number of image-space adjustable parameters is constrained as follows: (0<NPAR=NISAP=(NISAPR + NISAPC)<37). NISAPR is the number of image-space adjustable parameters that affect the image row-coordinate, and NISAPC the number that affect the image column-coordinate. If the basis option is on (APBASE=Y), specified image-space adjustable parameters are the basis RSM adjustable parameters. The total number of image-space adjustable parameters making up the basis set is constrained as follows: (0<NBASIS=NISAP=(NISAPR + NISAPC)<100).	2	BCS-A	N/A	1-36 (if APBASE=N) 1-99 (if APBASE=Y)	C
NISAPR	Number of Image-Space Adjustable Parameters for Image Row Coordinate. This field provides the total number of image-space adjustable parameters that adjust the image row coordinate. The general form for the row coordinate adjustment (Δr) corresponding to an adjustable parameter (ap) is as follows: $\Delta r = ap_{rijk} \cdot x^i \cdot y^j \cdot z^k$, where i,j,k are the corresponding powers of normalized Local coordinates x,y,z, respectively. Each adjustable parameter has units of pixels.	2	BCS-A	N/A	0-36 (if APBASE=N) 0-99 (if APBASE=Y)	C
…Begin for each image-space adjustable parameter for row adjustment (NISAPR entries)						
XPWRR	Row Parameter Power of X. The power (exponent) of x associated with this image-space adjustable parameter for image row adjustment. This power along with the following two powers (fields) uniquely specify the adjustable parameter. x is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image row coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C

YPWRR	Row Parameter Power of Y. The power (exponent) of y associated with this image-space adjustable parameter for image row adjustment. y is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image column coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
ZPWRR	Row Parameter Power of Z. The power (exponent) of z associated with this image-space adjustable parameter for image row adjustment. z is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized geodetic height coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
…End for each image-space adjustable parameter for row adjustment						
NISAPC	Number of Image-Space Adjustable Parameters for Image Column Coordinate. This field provides the total number of image-space adjustable parameters that adjust the image column coordinate. The general form for the column coordinate adjustment (Δc) corresponding to an adjustable parameter (ap) is as follows: $\Delta c = ap_{cijk} \cdot x^i \cdot y^j \cdot z^k$, where i,j,k are the corresponding powers of normalized Local coordinates x,y,z, respectively. Each adjustable parameter has units of pixels.	2	BCS-A	N/A	0-36 (if APBASE=N) 0-99 (if APBASE=Y)	C
…Begin for each image-space adjustable parameter for column adjustment (NISAPC entries)						
XPWRC	Column Parameter Power of X. The power (exponent) of x associated with this image-space adjustable parameter for image column adjustment. This power along with the following two powers (fields) uniquely specify the adjustable parameter. x is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image row coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
YPWRC	Column Parameter Power of Y. The power (exponent) of y associated with this image-space adjustable parameter for image column adjustment. y is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image column coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C

ZPWRC	Column Parameter Power of Z. The power (exponent) of z associated with this image-space adjustable parameter for image column adjustment. z is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized geodetic height coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
…End for each image-space adjustable parameter for column adjustment						
...end if (APTYP=I)						
"Ground-Space" Adjustable Parameters						
...if (APTYP=G)						

NGSAP	Number of Ground-Space Adjustable Parameters This field provides the total number of ground-space adjustable parameters. Each ground-space adjustable parameter is either associated with a "seven parameter" adjustment or is a first order "rate" term. The general form for the seven parameter adjustment is: $\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + \begin{bmatrix} \delta s & \delta\kappa & -\delta\beta \\ -\delta\kappa & \delta s & \delta\alpha \\ \delta\beta & -\delta\alpha & \delta s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix},$ where the vector on the left side of the equation is the ground-space adjustment in Local rectangular ground coordinates (meters), the vector on the far right side of the equation is the ground point location in normalized Local rectangular ground coordinates. The seven parameters $\delta x, \delta y, \delta z, \delta\alpha, \delta\beta, \delta\kappa, \delta s$, are termed x-offset, y-offset, z-offset, rotation angle alpha, rotation angle beta, rotation angle kappa, and scale. For identification purposes in the field below, these seven parameters are assigned 4 character identifications "OFFX", "OFFY" ,"OFFZ", "ROTX", "ROTY", "ROTZ", "SCAL", respectively. Each has units of meters. There a total of 9 possible rate terms $ap_{xx}, ap_{xy}, .., ap_{zz}$, termed "XRTX", "XRTY", "XRTZ", "YRTX", "YRTY", "YRTZ", "ZRTZ", "ZRTY, "ZRTZ, respectively. Their effect is illustrated as follows for the adjustable parameter "XRTY" (ap_{xy}) and corresponding adjustment Δx : $\Delta x = ap_{xy} y$. If the basis option is off (APBASE=N), specified ground-space adjustable parameters are the active RSM adjustable parameters. If the basis option is on (APBASE=Y), specified ground-space adjustable parameters are the basis RSM adjustable parameters. The total number of ground-space adjustable parameters (NGSAP) is constrained to be between 1 and 16 regardless the value of APBASE, i.e., regardless if the basis option is on or off. If the basis option is off, NPAR=NGSAP. If the basis option is on, NBASIS=NGSAP.	2	BCS-A	N/A	1-16	C
…Begin for each ground-space adjustable parameter (NGSAP entries)						

GSAPID	Ground-space Adjustable Parameter ID. This field identifies a ground-space adjustable parameter.	4	BCS-A	N/A	OFFX,OFFY,OFFZ, ROTX,ROTY,ROTZ, SCAL, XRTX, XRTY,XRTZ, YRTZ,YRTY,YRTZ, ZRTX,ZRTY,ZRTZ	C
…End for each ground-space adjustable parameter						
…end if (APTYP=G)						

...if (APBASE=Y)						
NBASIS	Number of Basis Adjustable Parameters. This field contains the number of RSM adjustable parameters in the basis set. It is equal to the number of columns in the matrix A. NBASIS=NISAP or NGSAP, depending on whether the previously identified adjustable parameters were "image-space" or "ground-space" adjustable parameters. The number of columns must be no less than the number of rows in the matrix A, i.e., NBASIS $\geq$ NPAR. The size of the matrix A is also constrained such that NPAR*NBASIS ≤ 1296.	2	BCS-N	N/A	1-99	C
...Begin for each A element (NPAR*NBASIS entries)						
AEL	Matrix A Element. This field contains an element of the matrix A. The elements are stored in row major order.	21	BCS-A	N/A	±9.99999999999999E±99	C
...End loop over elements of matrix A						
...end if (APBASE=Y)						
...end if (INCAPD=Y)						
Direct Error Covariance (Cross-covariance blocks for the associated image with each image listed (in order) in field IIDI)						
....Begin for each image (NIMGE entries)						
…Begin for each cross-covariance matrix element (NROWCB*NCOLCB) entries)						
CRSCOV	Cross-covariance Element. This field contains an element of the cross-covariance matrix (block) for the associated image and the image. The matrix dimension is NROWCBxNCOLCB. Elements are stored in row major order.	21	BCS-A	N/A	±9.99999999999999E±99 Element values must correspond to a symmetric (auto) covariance matrix but not necessarily a symmetric cross-covariance matrix. Element values must be consistent with an assembled positive (semi) definite direct error covariance.	R
…End for each element						
...End for each image						

Table 1: RSMDCX TRE format table

2.0 RSM Adjustable Parameters (RSMAPX) TRE

2.1 Overview

The Replacement Sensor Model Adjustable Parameters TRE (RSMAPX) identifies the current values of active RSM adjustable parameters for the associated image. If the RSMAPX TRE is not provided, all RSM adjustable parameters for the associated image are assumed to have a value of zero. When the RSMAPX TRE is provided, corresponding values typically reflect the output of an RSM adjustment process (e.g. triangulation).

2.2 Active adjustable parameter values

This TRE identifies the active RSM adjustable parameters for the associated image, as detailed in section 2.3. Their corresponding values are also stored in the RSM Adjustment Vector (R) contained in this TRE (field PARVAL). The order of the elements in the RSM Adjustment Vector correspond to the exact order the active adjustable parameters are identified in this populated TRE (see Table 2). The dimension of the RSM Adjustment Vector equals the number of active adjustable parameters (field NPAR). The RSM Adjustment Vector is used to adjust the RSM ground-to-image function, as detailed in section 2.3.

2.3 Adjustable parameter definitions and identification

Active RSM adjustable parameters for the associated image are either active RSM "image-space" adjustable parameters or active RSM "ground-space" adjustable parameters, as specified by field APTYP in the RSMAPX TRE.

RSM "image-space" adjustable parameters correspond to adjustable parameters that adjust an image row coordinate value (r) and an image column coordinate value (c) corresponding to an arbitrary ground point location $X = [x \quad y \quad z]^T$. An individual adjustable parameter either adjusts an image row coordinate value or an image column coordinate value. The adjustments Δr and Δc corresponding to adjustable parameters ap_{rijk} and ap_{cijk} are computed as follows:

$$\Delta r = ap_{rijk} x^i y^j z^k$$

$$\Delta c = ap_{cijk} x^i y^j z^k$$

The adjustable parameters (ap_{rijk} and ap_{cijk}) are uniquely identified by their collective x, y, z powers (exponents) and whether they adjust image row or image column coordinates. The coordinates x, y, and z correspond to normalized ground point coordinates expressed in a Local coordinate system. Normalization is performed by an offset and scale factor for each coordinate. These normalization parameters are in contiguous fields (NSFX-NOFFZ), and allow for an approximate range of (-1,1) for each ground coordinate value. An example of their application for normalization of the y coordinate is as follows:

$$y \to (y - offset_y)/scale_y \ .$$

Because the ground coordinates are normalized, all "image-space" adjustable parameters have units of pixels, as do the corrections Δr and Δc.

Normalization of the Local coordinates helps to insure overall stability since the value of $x^i y^j z^k$ that multiplies an adjustable parameter during an image row or column adjustment can become extremely large if coordinates are not normalized for large images and large exponents.

There are two possible choices for the Local coordinate system, either Local Rectangular or Local Non-Rectangular, as specified in field LOCTYP. For Local Non-Rectangular, x, y, and z correspond to the ground point's corresponding image row coordinate, image column coordinate, and geodetic height, respectively. The Local Rectangular coordinate system is defined as a rectangular system that is offset and rotated relative to the WGS-84 coordinate system. It is typically specified as a local tangent plane system centered within the RSM image domain’s footprint at a nominal height above the ellipsoid and rotated to be aligned as follows: the z-axis is aligned with the imaging locus direction (line-of-sight vector for an electro-optical sensor), the x-axis is aligned with the image line (“sweep” or "scan") direction, and the y-axis completes a right-handed rectangular system. (When the Local Rectangular coordinate system is footprint centered, corresponding Local Rectangular coordinate normalization offsets, such as $offset_y$, typically have a value of zero.)

Figure 3 illustrates a typical Local Rectangular coordinate system. Specification of a Local Rectangular coordinate system is unique to the associated image and based on contiguous fields (XUOL-ZUZL) as detailed later in this introduction.

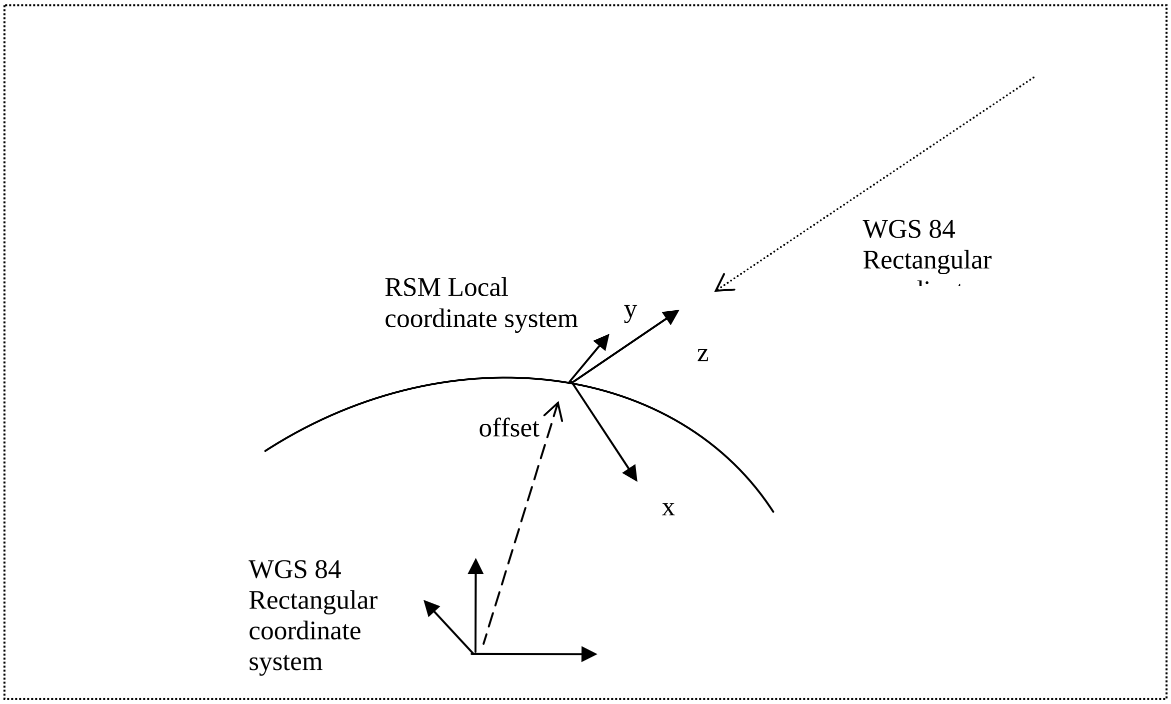

Figure 3: Example of RSM Local Rectangular Coordinate System

Note that the choice of Local Rectangular or Local Non-rectangular is provided for flexibility. The Local Rectangular coordinate system inherits general analytic advantages associated with rectangular (orthonormal) coordinates, and its absolute orientation is insensitive to any abrupt changes in imaging geometry across the imaging time interval. The Local Non-Rectangular coordinate system may provide advantages when very long images are (smoothly) scanned due to significant changes in instantaneous image geometry from one end of the image to the other. The coordinate system is continuously in alignment with these changes.

RSM "ground-space" adjustable parameters reference normalized Local Rectangular coordinates only. The coordinate system is typically specified as a local tangent plane system centered within the RSM image domain's footprint at a nominal height above the ellipsoid (z-axis vertical). An individual "ground-space" adjustable parameter is either a parameter associated with a "seven parameter" adjustment or a "rate" adjustment. The seven parameter adjustment is defined as follows, where the symbols $\{\delta x \quad \delta y \quad \delta z \quad \delta\alpha \quad \delta\beta \quad \delta\kappa \quad \delta s\}$ correspond to the adjustable parameters:

$$\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix}=\begin{bmatrix}\delta x\\ \delta y\\ \delta z\end{bmatrix}+\begin{bmatrix}\delta s & \delta\kappa & -\delta\beta\\ -\delta\kappa & \delta s & \delta\alpha\\ \delta\beta & -\delta\alpha & \delta s\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

The vector on the left side of the above equation corresponds to adjustments in the un-normalized coordinates of the ground point expressed in Local Rectangular coordinates with units of meters. The vector on the far right side of the equation corresponds to normalized coordinates of the ground point expressed in Local Rectangular coordinates. Because these coordinates are unit-less, the adjustable parameters all have units of meters, as do the corrections $\Delta x, \Delta y, \Delta z$.

(If Local coordinate scale factors ($scale_x, scale_y, scale_z$) are set equal in value by the TRE generation process, Local coordinate values no longer necessarily range from -1.0 to 1.0. However, the above seven parameter adjustment is now equivalent to the standard photogrammetric seven parameter (small angle) transformation. It is recommended that the scale factors be set equal to a common value in a manner that yields ranges for the three local coordinates as close as possible to interval -1.0 to 1.0.)

There are nine possible ground-space adjustable parameters corresponding to rate adjustments and denoted by the symbols $\{ap_{xx}, ap_{xy}, .., ap_{zz}\}$. They adjust the un-normalized coordinates of the ground point in Local Rectangular coordinates specifically as follows:

$\Delta x = ap_{xx}x$, $\Delta x = ap_{xy}y$, $\Delta x = ap_{xz}z$,

$\Delta y = ap_{yx}x$, $\Delta y = ap_{yy}y$, $\Delta y = ap_{yz}z$,

$\Delta z = ap_{zx}x$, $\Delta z = ap_{zy}y$, $\Delta z = ap_{zz}z$.

Again, these adjustable parameters and the corrections have units of meters.

Each of the 16 possible ground-space adjustable parameters is identified by a unique four character identifier detailed in the TREs specified format (Table 2).

Note that application of RSM adjustable parameters, whether "image-space" or "ground-space" adjustable parameters, first requires converting the corresponding ground point from representation in the RSM primary ground coordinate system to the appropriate Local system. And for the case of "ground-space" adjustable parameters, the adjusted ground point must also be converted back to the RSM primary coordinate system.

Figure 4 presents the RSM adjustable ground-to-image function $h(X,R)$, where X corresponds to the un-normalized three dimensional ground point in the RSM primary ground coordinate system. The functions $I\_adj(X,R)$ and $X\_adj(X,R)$ apply the previously documented adjustment equations for active "image-space" and "ground-space" adjustable parameters, respectively. (The functions also internally convert X from the primary system to the (normalized) Local system.)

$\Delta I = [\Delta r \quad \Delta c]^T$ denotes the summed effects at ground point location X of all active RSM "image-space" adjustable parameters. For example, if the active "image-space" adjustable parameters correspond to (combined) powers in x and y less than or equal to one: $\Delta r = ap_{r000} + ap_{r100} \cdot x + ap_{r010} \cdot y$, and $\Delta c = ap_{c000} + ap_{c100} \cdot x + ap_{c010} \cdot y$.

$\Delta X = [\Delta x \quad \Delta y \quad \Delta z]^T$ denotes the summed effects at ground point location X of all active RSM "ground-space" adjustable parameters.

The vector R represents the active RSM adjustable parameters (values) in the order that they are specified in this populated TRE, e.g., vector element two corresponds to the second active adjustable parameter identified in the populated TRE (see Table 2). (Internally, the RSM ground-to-image function $g(X)$ is actually performed with respect to normalized coordinates. The RSM ground-to-image function handles all required normalization and un-normalization, as described in the RSMPCA TRE and the RSMGGA TRE.)

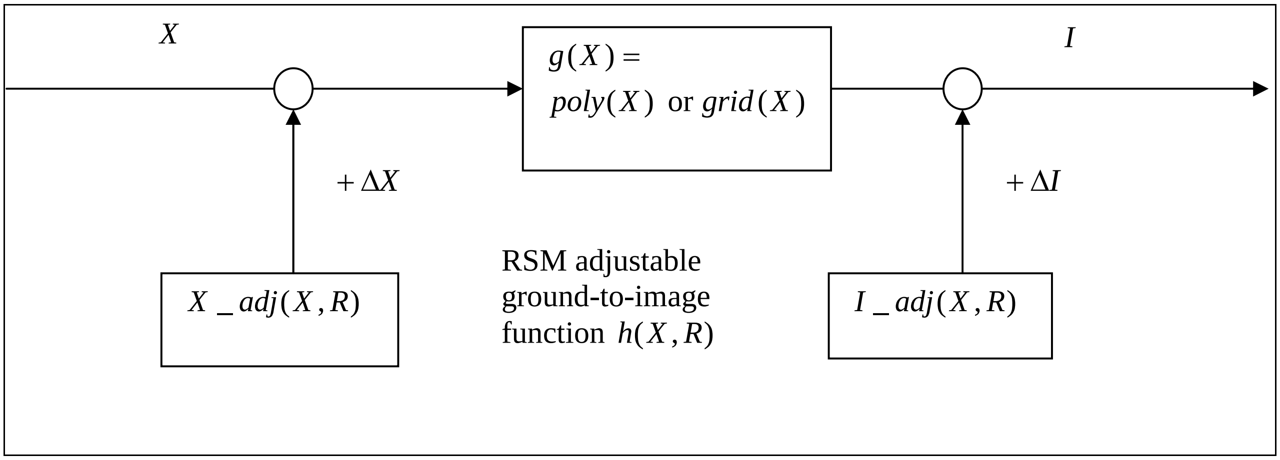

Figure 4: RSM adjustable ground-to-image function

The total number of active "image-space" adjustable parameters is specified in field (NISAP). Individual active adjustable parameters are identified in contiguous fields (XPWRR-ZPWRR) and contiguous fields (XPWRC-ZPWRC).

The total number of active "ground-space" adjustable parameters is specified in field (NGSAP). Individual active parameters are identified in field (GSAPID).

2.3.1 Basis Option

When this option is invoked (APBASE=Y), the set of RSM adjustable parameters specified in this TRE (as described previously) become a "basis" set of adjustable parameters. Symbolically, they are contained in the vector R, assumed to have n elements. Another set of RSM adjustable parameters is defined as a linear combination of the elements of R. Symbolically, this new set is contained in the vector R', where $R' = AR$, and the matrix A is $m \times n$, $m \leq n$, with the rank of A equal to m. The vector R' contains the (new) set of active adjustable parameters.

The RSM Adjustment Vector contained in this TRE is now defined as R'. The field NPAR now corresponds to the number of elements (m) in R'. The field NBASIS corresponds to the number of elements (n) in R.

Typically, the matrix A is determined during initial generation of this TRE along with a corresponding error covariance using principal component analysis (see RSMDCX for an overview of the process and its potential benefits). R is also a linear combination of R' based on the pseudo-inverse of A, designated as $A^{\#}$ (or AI). Thus, $R = A^{\#} R'$, where $A^{\#} = A^T (AA^T)^{-1} = A^T$.

When this option is on, the RSMAPX TRE contains the identification of the adjustable parameters that make up the elements of the basis (R), the matrix A that maps the basis (R) to the set of active adjustable parameters (R'), and the values of the active adjustable parameters, i.e., the (new) RSM Adjustment

Vector (R'). In order to adjust the RSM ground-to-image function, an RSM exploiter maps the current values of R' contained in this TRE to current values of R via $R = A^T R'$. It then applies R to the ground-to-image function as illustrated in Figure 4.

2.3.2 Local Rectangular coordinate system details

The following defines the Local Rectangular coordinate system relative to the WGS 84 Rectangular coordinate system. The contiguous fields XUOL through ZUOL specify the origin (offset) of the Local coordinate system relative to the WGS 84 Rectangular coordinate system, and the contiguous fields XUXL through through ZUZL elements of the rotation matrix. These fields are provided in this TRE.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{LOCAL}} = \begin{bmatrix} XUXL & YUXL & ZUXL \\ XUYL & YUYL & ZUYL \\ XUZL & YUZL & ZUZL \end{bmatrix} \cdot \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{WGS}-84} - \begin{bmatrix} XUOL \\ YUOL \\ ZUOL \end{bmatrix} \right).$$

Note that the definition of the Local Rectangular coordinate system is also redundantly supplied in other TREs for the associated image. Also, in order to convert a ground point X represented in the RSM primary ground coordinate system (e.g., Geodetic) to the Local Rectangular coordinate system, it must first be converted from the RSM primary system to the WGS 84 Rectangular coordinate system.

2.4 RSMAPX format

Table 2 specifies the detailed format for the Replacement Sensor Model Adjustable Parameters (RSMAPX) TRE.

RSMAPX – Replacement Sensor Model Adjustable Parameters

Field	Name/Description	Size	Format	Units	Estimated Value Range	Type
TAG Information						
CETAG	<u>Unique Extension Type Identifier</u> Unique TRE identifier	6	BCS-A	N/A	RSMAPX	R
CEL	<u>Length of User-Defined Data</u> Length in bytes of data contained in subsequent fields. (TREs length is 11 plus the value given in the CEL field)	5	BCS-N	bytes	00321 to 28411	R
Image information						

IID	Image Identifier. This field contains a character string that uniquely identifies the original full image that corresponds to the associated image. This is not to be confused with the identification of an image derived by filtering, chipping, re-sampling, or other such image to image transformations. The image identifier is left justified with trailing spaces.	80	BCS-A	N/A	N/A All spaces if unavailable	<R>
EDITION	RSM Image Support Data Edition. This field contains a character string that uniquely identifies the RSM support data for the associated original full image. It is to consist of an identifier of up to 20 characters for the processor that generated the RSM support data, to which is appended up to 20 characters that are unique to that processor.	40	BCS-A	N/A	N/A	R
TID	Triangulation ID. This field contains an identifier that is unique to the most recent process after RSM support data generation that led to the adjustments and/or error covariance in this RSM support data edition.	40	BCS-A	N/A	N/A	R
RSM Adjustable Parameter Identification for the associated image						
NPAR	Number of Active RSM Adjustable Parameters. This field contains the total number of (active) RSM adjustable parameters for the associated image. It is the number of elements (components) in the RSM Adjustment Vector (field PARVAL). The value is constrained to less than 37 to insure an RSM Adjustment Vector (and associated error covariance) of practical size. (If the "basis" option is off (APBASE=N), NPAR=NISAP if APTYP=I, and NPAR=NGSAP if APTYP=G. If the basis option is on (APBASE=Y), NPAR corresponds to the number of (new) active adjustable parameters and the number of rows in the Matrix A, as described for field APBASE.)	2	BCS-N	N/A	01 to 36	R
APTYP	Adjustable Parameter Type. This field identifies whether RSM adjustable parameters are "image-space" (APTYP=I) or "ground-space" (APTYP=G) adjustable parameters.	1	BCS-A	N/A	I or G	R
LOCTYP	Local Coordinate System Identifier. The field identifies whether the Local coordinate system references rectangular ground coordinates (LOCTYP=R) or non-rectangular (image row/image column/geodetic height) coordinates (LOCTYP=N). If RSM adjustable parameters are specified as "ground-space" (APTYP=G), the only valid value is LOCTYP=R.	1	BCS-A	N/A	R or N	R
Normalization Factors for the Local System						
NSFX	Normalization Scale Factor for X. This field contains the normalization scale factor for the x component of the Local coordinate system Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	R

NSFY	Normalization Scale Factor for Y. This field contains the normalization scale factor for the y component of the Local coordinate system Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	R
NSFZ	Normalization Scale Factor for Z. This field contains the normalization scale factor for the z component of the Local coordinate system	21	BCS-A	meters	±9.99999999999999E±99	R
NOFFX	Normalization Offset for X. This field contains the normalization offset for the x component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	R
NOFFY	Normalization Offset for Y. This field contains the normalization offset for the y component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	R
NOFFZ	Normalization Offset for Z. This field contains the normalization offset for the z component of the Local coordinate system.	21	BCS-A	meters	±9.99999999999999E±99	R
Local Rectangular Ground Coordinates Detailed Definition for Associated Image						
...if (LOCTYP=R)						
XUOL	Local Coordinate Origin (XUOR). This field provides the WGS 84 *X* coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E+99	R
YUOL	Local Coordinate Origin (YUOL). This field provides the WGS 84 *Y* coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E+99	R
ZUOL	Local Coordinate Origin (ZUOL). This field provides the WGS 84 *Z* coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E+99	R
XUXL	Local Coordinate Unit Vector (XUXL). This field provides the WGS 84 *X* component of the unit vector defining the *X*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
XUYL	Local Coordinate Unit Vector (XUYL). This field provides the WGS 84 *X* component of the unit vector defining the *Y*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
XUZL	Local Coordinate Unit Vector (XUZL). This field provides the WGS 84 *X* component of the unit vector defining the *Z*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R

YUXL	Local Coordinate Unit Vector (YUXL). This field provides the WGS 84 *Y* component of the unit vector defining the *X*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
YUYL	Local Coordinate Unit Vector (YUYL). This field provides the WGS 84 *Y* component of the unit vector defining the *Y*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
YUZL	Local Coordinate Unit Vector (YUZL). This field provides the WGS 84 *Y* component of the unit vector defining the *Z*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
ZUXL	Local Coordinate Unit Vector (ZUXL). This field provides the WGS 84 *Z* component of the unit vector defining the *X*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
ZUYL	Local Coordinate Unit Vector (ZUYL). This field provides the WGS 84 *Z* component of the unit vector defining the *Y*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
ZUZL	Local Coordinate Unit Vector (ZUZL). This field provides the WGS 84 *Z* component of the unit vector defining the *Z*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	R
...end if (LOCTYP=R)						
RSM Adjustable Parameter Basis Option						

APBASE	Basis Option. This field indicates whether the RSM adjustable parameters "basis" option is on (APBASE=Y). If this option is off (APBASE=N), the RSM adjustable parameters specified in the following fields are the active RSM adjustable parameters. The order (component number) of an active RSM adjustable parameter is the order in which it is specified. If this option is on, the RSM adjustable parameters specified in the following fields are the basis set of RSM adjustable parameters. The order (component number) of a basis RSM adjustable parameter is the order in which it is specified. If this option is on, the active RSM adjustable parameters are a linear combination of the basis set of RSM adjustable parameters. The matrix A (field AEL) maps the basis set to the active set of RSM adjustable parameters. In addition, the pseudo-inverse of the matrix A is equal to the matrix A transpose. It maps the active set to the basis set of RSM adjustable parameters. The matrix A is mxn, where $m \le n$ and the rank of A equals m. The number of adjustable parameters (n) in the basis set is specified in field NBASIS. The number of active adjustable parameters (m) is specified in the field NPAR. The RSM image error covariance is always with respect to the active RSM adjustable parameters. For example, the second active RSM adjustable parameter corresponds to row 2 and column 2 of the image (auto) covariance, and corresponds to row 2 and column k of the cross-covariance of the associated image with image k.	1	BCS-A	NA	Y or N	C
"Image-Space" Adjustable Parameters						
...if (APTYP=I)						

NISAP	Number of Image-Space Adjustable Parameters. This field contains the total number of image-space adjustable parameters. If the basis option is off (APBASE=N), specified image-space adjustable parameters are the active RSM adjustable parameters. The total number of image-space adjustable parameters is constrained as follows: (0<NPAR=NISAP=(NISAPR + NISAPC)<37). NISAPR is the number of image-space adjustable parameters that affect the image row-coordinate, and NISAPC the number that affect the image column-coordinate. If the basis option is on (APBASE=Y), specified image-space adjustable parameters are the basis RSM adjustable parameters. The total number of image-space adjustable parameters making up the basis set is constrained as follows: (0<NBASIS=NISAP=(NISAPR + NISAPC)<100).	2	BCS-A	N/A	1-36 (if APBASE=N) 1-99 (if APBASE=Y)	C
NISAPR	Number of Image-Space Adjustable Parameters for Image Row Coordinate. This field provides the total number of image-space adjustable parameters that adjust the image row coordinate The general form for the row coordinate adjustment (Δr) corresponding to an adjustable parameter (ap) is as follows: $\Delta r = ap_{rijk} \cdot x^i \cdot y^j \cdot z^k$, where i,j,k are the corresponding powers of normalized Local coordinates x,y,z, respectively. Each adjustable parameter has units of pixels.	2	BCS-A	N/A	0-36 (if APBASE=N) 0-99 (if APBASE=Y)	C
…Begin for each image-space adjustable parameter for row adjustment (NISAPR entries)						
XPWRR	Row Parameter Power of X. The power (exponent) of x associated with this image-space adjustable parameter for image row adjustment. This power along with the following two powers (fields) uniquely specify the adjustable parameter. x is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image row coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
YPWRR	Row Parameter Power of Y. The power (exponent) of y associated with this image-space adjustable parameter for image row adjustment. y is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image column coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C

ZPWRR	Row Parameter Power of Z. The power (exponent) of z associated with this image-space adjustable parameter for image row adjustment. z is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized geodetic height coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
...End for each active image-space adjustable parameter for row adjustment						
NISAPC	Number of Image-Space Adjustable Parameters for Image Column Coordinate. This field provides the total number of image-space adjustable parameters that adjust the image column coordinate. The general form for the column coordinate adjustment (Δc) corresponding to an adjustable parameter (ap) is as follows: $\Delta c = ap_{cijk} \cdot x^i \cdot y^j \cdot z^k$, where i,j,k are the corresponding powers of normalized Local coordinates x,y,z, respectively. Each adjustable parameter has units of pixels.	2	BCS-A	N/A	0-36 (if APBASE=F) 0-99 (if APBASE=T)	C
…Begin for each image-space adjustable parameter for column adjustment (NISAPC entries)						
XPWRC	Column Parameter Power of X. The power (exponent) of x associated with this image-space adjustable parameter for image column adjustment. This power along with the following two powers (fields) uniquely specify the adjustable parameter. x is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image row coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
YPWRC	Column Parameter Power of Y. The power (exponent) of y associated with this image-space adjustable parameter for image column adjustment. y is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image column coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
ZPWRC	Column Parameter Power of Z. The power (exponent) of z associated with this image-space adjustable parameter for image column adjustment. z is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized geodetic height coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
...End for each image-space adjustable parameter for column adjustment						
...end if (APTYP=I)						
"Ground-Space" Adjustable Parameters						
...if(APTYP=G)						

NGSAP	Number of Ground-Space Adjustable Parameters This field provides the total number of ground-space adjustable parameters. Each ground-space adjustable parameter is either associated with a "seven parameter" adjustment or is a first order "rate" term. The general form for the seven parameter adjustment is: $\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + \begin{bmatrix} \delta s & \delta\kappa & -\delta\beta \\ -\delta\kappa & \delta s & \delta\alpha \\ \delta\beta & -\delta\alpha & \delta s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix},$ where the vector on the left side of the equation is the ground-space adjustment in Local rectangular ground coordinates (meters), the vector on the far right side of the equation is the ground point location in normalized Local rectangular ground coordinates. The seven parameters $\delta x, \delta y, \delta z, \delta\alpha, \delta\beta, \delta\kappa, \delta s$, are termed x-offset, y-offset, z-offset, rotation angle alpha, rotation angle beta, rotation angle kappa, and scale. For identification purposes in the field below, these seven parameters are assigned 4 character identifications "OFFX", "OFFY" ,"OFFZ", "ROTX", "ROTY", "ROTZ", "SCAL", respectively. Each has units of meters. There a total of 9 possible rate terms $ap_{xx}, ap_{xy}, .., ap_{zz}$, termed "XRTX", "XRTY", "XRTZ", "YRTX", "YRTY", "YRTZ", "ZRTZ", "ZRTY, "ZRTZ, respectively. Their effect is illustrated as follows for the adjustable parameter "XRTY" (ap_{xy}) and corresponding adjustment Δx : $\Delta x = ap_{xy} y$. If the basis option is off (APBASE=N), specified ground-space adjustable parameters are the active RSM adjustable parameters. If the basis option is on (APBASE=Y), specified ground-space adjustable parameters are the basis RSM adjustable parameters. The total number of ground-space adjustable parameters (NGSAP) is constrained to be between 1 and 16 regardless the value of APBASE, i.e., regardless if the basis option is on or off. If the basis option is off, NPAR=NGSAP. If the basis option is on, NBASIS=NGSAP.	2	BCS-A	N/A	1-16	C
…Begin for each ground-space adjustable parameter (NGSAP entries)						

GSAPID	Ground-space Adjustable Parameter ID. This field identifies a ground-space adjustable parameter.	4	BCS-A	N/A	OFFX,OFFY,OFFZ, ROTX,ROTY,ROTZ, SCAL, XRTX, XRTY,XRTZ, YRTZ,YRTY,YRTZ, ZRTX,ZRTY,ZRTZ	C
...End for each ground-space adjustable parameter						
...end if (APTYP=G)						
...if (APBASE=Y)						
NBASIS	Number of Basis Adjustable Parameters. This field contains the number of RSM adjustable parameters in the basis set. It is equal to the number of columns in the matrix A. NBASIS=NISAP or NGSAP, depending on whether the previously identified adjustable parameters were "image-space" or "ground-space" adjustable parameters. The number of columns must be no less than the number of rows in the matrix A, i.e., NBASIS$\geq$NPAR. The size of the matrix A is also constrained such that NPAR*NBASIS≤ 1296.		BCS-N	N/A	1-99	C
...Begin for each A element (NPAR*NBASIS entries)						
AEL	Matrix A Element. This field contains an element of the matrix A. The elements are stored in row major order.	21	BCS-A	N/A	±9.99999999999999E±99	C
...End loop over elements of matrix A						
...end if (APBASE=Y)						
RSM Adjustment Vector values						
…Begin for each RSM Adjustment Vector component for the associated image (NPAR entries)						
PARVAL	Component Value. This field contains the value contained in the next component of the RSM Adjustable Parameter Vector.	21	BCS-A	N/A	±9.99999999999999E+99	R
…End for each component						

Table 2: RSMAPX TRE format table

3.0 RSM Indirect Error Covariance (RSMECX) TRE

3.1 Overview

The Replacement Sensor Model Error Covariance TRE (RSMECX) contains general error covariance information applicable to correlated images from the same sensor. This information includes the error (auto) covariance for the associated image and a temporal correlation model used to build the error cross-covariance between images. The corresponding multi-image error covariance generated from this information is termed the RSM indirect error covariance.

Errors correspond to a specified set(s) of active RSM adjustable parameters. Note that in general, the RSM indirect error covariance provides a statistical description of image support data error for one or more images.

The indirect error covariance can correspond to an arbitrary number of pixel locations (times) from an arbitrary number of images from the same sensor. (All the images have different original full image IDs.) In order to generate an indirect error covariance associated with multiple images, an RSMECX TRE from each image must be used.

If there are p_i pixel locations of interest and m_i active RSM adjustable parameters for image i, $i = 1,..,n$, the indirect error covariance generated will be an $\sum_{i=1}^{n} p_i \cdot m_i \times \sum_{i=1}^{n} p_i \cdot m_i$ matrix. All images must correspond to the same sensor, but the set (identity) of active RSM adjustable parameters can vary from image to image in order to best represent the original sensor model's adjustability and error covariance across different imaging geometries. The set (identity) of adjustable parameters for the original sensor model is assumed invariant across the images.

The RSM indirect error covariance is relative to errors in the active RSM adjustable parameter values. The actual values of the RSM adjustable parameters are provided in the RSMAPX TRE for each image involved. For a given image, if the corresponding RSMAPX TRE is not provided, the corresponding RSM adjustable parameter values are assumed equal to zero, corresponding to unadjusted RSM image support data.

The indirect error covariance is typically applicable when image support data (original or RSM) is unadjusted, i.e., not the result of a triangulation. If a triangulation was performed, the direct error covariance (RSMDCX) is typically used to represent the error covariance. If both an indirect error covariance (RSMECX) and a direct error covariance (RSMDCX) are in the RSM TRE set for the associated image, the direct error covariance takes precedence. If in addition, an RSM adjustment has been performed as indicated by the presence of RSMAPX, the direct error covariance pertains to the adjusted RSM image support data and the indirect error covariance pertains to the unadjusted (a priori) RSM image support data.

3.2 Groups of error covariance information

The RSMECX TRE contains the following specific groups of information required to construct the indirect error covariance. If the indirect error covariance is applicable to other images in addition to the associated image, the RSMECX TRE from each of these images is also required. The groups of information are:

1. Error (auto) covariance relative to the original sensor model adjustable parameters applicable at an arbitrary time (pixel location) in the associated image's RSM image domain. An error covariance is actually supplied for each independent subset of original adjustable parameters. These error covariances are block diagonals within the full error covariance (with block zero's elsewhere). The field NUMOPG specifies the number of original adjustable parameters and the field ERRCVG specifies the actual error covariance element values for an independent subset. Note that this data does not actually describe (or rely on a description of) the original sensor model adjustable parameters. Thus, no knowledge of the original sensor model is provided or needed for successful RSMECX TRE implementation. (Of course, knowledge of the original sensor model is required for RSMECX TRE generation prior to its dissemination via the RSM image support data.) The membership of the (unknown) adjustable parameters within each independent subgroup is assumed invariant across the correlated images. There are a total of NPARO original adjustable parameters for the associated image.
2. A time correlation model for the above errors that allows for generation of the cross-covariance of original sensor model adjustable parameter errors at two different times. There are multiple time correlation models, each associated with a different, independent subset of original sensor model adjustable parameters. All of these models have a common form – a piece-wise linear, non-negative, convex function. The function has a "starting" correlation value of one at tau equal to zero, and has a correlation "floor" value of zero for large values of tau. Tau (τ) is the correlation function's independent variable, a time difference for this particular application. The fields TCDF, NCSEG, CORSEG, and TAUSEG define the correlation function for an independent subset of original adjustable parameters. For a given independent subset, the corresponding value of TCDF is assumed invariant across the correlated images.
3. Identification of the applicable (active) RSM adjustable parameters and their index into the RSM error cross-covariance for the associated image. There are a total of NPAR active RSM adjustable parameters for the associated image. The RSM error cross-covariance is applicable to errors in the values of the active adjustable parameters, and is described in more detail below. Also, the definition of the ground coordinate system referenced by the RSM adjustable parameters is provided. Note that the identity of the applicable active RSM adjustable parameters can vary from image to image from the same sensor. If they do, they typically vary +/- 2 elements from a common set of (sensor dependent) adjustable parameters.

4. A mapping matrix for the associated image that relates the error covariance associated with the original sensor model adjustable parameters to an error covariance associated with the active RSM adjustable parameters. Field MAP contains the value of the mapping matrix elements. Note that the mapping matrix is different in value (and possibly row dimension) for each correlated image. The mapping matrix has NPAR rows and NPARO columns.

3.3 Indirect error covariance form

The following further describes and integrates the above groups (1-4) of indirect error covariance information. Assume a total of n correlated images from the same sensor. Assume p_i pixel locations (times) of interest and m_i active RSM adjustable parameters for image i, $i = 1,..,n$. Define $q_i = p_i \cdot m_i$, and $q = \sum_{i=1}^{n} q_i$. The indirect error covariance CR is the following $q \times q$ symmetric matrix:

$$CR = \begin{bmatrix} C_{R11} & C_{R12} & . & . & C_{R1q} \\ . & C_{R22} & . & . & . \\ . & . & . & C_{Rij} & . \\ . & . & . & . & . \\ . & . & . & . & C_{Rqq} \end{bmatrix}.$$

C_{Rij} is the cross-covariance (block) between the errors in the active RSM adjustable parameters at the time of pixel i $(i = 1,..,q)$ and at the time of pixel j $(j = 1,..,q)$. It has dimension $m_{i*} \times m_{j*}$, where $i*$ is the image that contains pixel i, and $j*$ is the image that contains pixel j. C_{Rij} is computed by an RSM exploiter as follows:

$$C_{Rij} = \Phi_{i*} C_{Sij} \Phi_{j*}^{T},$$

where Φ_{i*} is the mapping matrix corresponding to image $i*$. C_{Sij} is the cross-covariance between the errors in the original sensor model adjustable parameters at the time of pixel i and at the time of pixel j. If there are s original adjustable parameters per image, C_{Sij} is an sxs cross-covariance matrix, and Φ_{i*} is an $m_{i*} \times s$ mapping matrix. (Note that when $i = j$, the RSM error cross-covariance C_{Rij} becomes the RSM error (auto) covariance C_{Rii} for the active RSM adjustable parameters at the time of pixel i.)

As mentioned previously, C_{Rij} is applicable to both the active RSM adjustable parameters for image $i*$ and the active RSM adjustable parameters for image $j*$ The row index into C_{Rij} corresponds to the order in which the active adjustable parameters are identified in image $i*$'s populated RSMECX TRE (see Table 3). For example, if an RSM adjustable parameter is the third active adjustable parameter identified in the TRE, row 3 of C_{Rij} corresponds to the error in the value of that active adjustable parameter applicable at the time of pixel i. Similarly, the column index into C_{Rij} corresponds to the order in which the active adjustable parameters are identified in image $j*$'s populated RSMECX TRE.

C_{Sij}, used in the computation of C_{Rij}, is computed from the original sensor model adjustable parameter error covariance C_{Si*} associated with image $i*$, the corresponding time correlation function $\rho_{Si*}(\tau)$ associated with image $i*$, and the (absolute) time difference τ between pixels i and j:

$$C_{Sij} = \rho_{Si*}(\tau)C_{Si*} = \begin{bmatrix} \rho_{Si*1}(\tau)C_{Si*1} & 0 & . & . & 0 \\ . & . & . & . & . \\ . & . & \rho_{Si*k}(\tau)C_{Si*k} & . & . \\ . & . & . & . & 0 \\ . & . & . & . & \rho_{Si*w}(\tau)C_{Si*w} \end{bmatrix}.$$

The symmetric C_{Sij} is in block diagonal form. The individual C_{Si*k} and $\rho_{Si*k}(\tau)$ correspond to the error covariance and scalar time correlation function for the independent subgroup k of original adjustable parameters. Since $\rho_{Si*k}(\tau)$ is a scalar function of τ, if C_{Si*k} is $s_k \times s_k$, then $\rho_{Si*k}(\tau)C_{Si*k}$ is an $s_k \times s_k$ diagonal block of C_{Sij}. A total of w subgroups is assumed, thus $s = \sum_{k=1}^{w} s_k$.

The above formulation for C_{Sij}, and hence C_{Rij}, assumes that C_{Si*k} and $\rho_{Si*k}(\tau)$ are invariant across images ($i*$) when multiple images are involved. This corresponds to a wide-sense stationary stochastic error model and is applicable to most applications involving the use of the RSMECX TRE. If instead, a higher fidelity non-stationary stochastic error model is applicable as indicated by variable values of C_{Si*k} and/or $\rho_{Si*k}(\tau)$ across the images, additional processing is required in order to ensure a valid (positive semi-definite) indirect error covariance CR. The additional processing is detailed later in this RSMECX TRE description.

(Note that, in general, all errors (ε) referenced in the various RSM TRE descriptions are assumed unbiased, i.e. $E\{\varepsilon\} = 0$. In addition, the term "independent subgroups" of adjustable parameters actually refers to uncorrelated errors associated with adjustable parameters from the different subgroups, i.e. $E\{\varepsilon 1 \cdot \varepsilon 2\} = 0$, where $\varepsilon 1$ represents the error in an adjustable parameter from independent subgroup 1, and $\varepsilon 2$ represents the error in an adjustable parameter from independent subgroup 2.)

As mentioned previously, the fields TCDF, NCSEG, CORSEG, and TAUSEG define the correlation function for an independent subset (subgroup) of original adjustable parameters. There are multiple linear segments i ($i = 1,..N$) associated with a correlation function, as specified by the value of N (NCSEG). Each of these segments has a corresponding correlation value ρ_i (CORSEG) and correlation time value τ_i (TAUSEG) applicable at the beginning of the segment. Thus, the value of a correlation function $\rho(\tau)$ (e.g. $\rho_{Si*k}(\tau)$) for a given value of τ is as follows (see Figure 5):

$$\rho(\tau) = \begin{cases} \rho_i + \dfrac{(\rho_{i+1} - \rho_i)(\tau - \tau_i)}{(\tau_{i+1} - \tau_i)} & , \tau_i \le \tau < \tau_{i+1} \\ 0 & , \tau_N \le \tau \end{cases} .$$

Note that τ_N is also termed the "cut-off" time, or τc.

The above equation is applicable to original adjustable parameter errors modeled as "image element" errors, as specified by a value of 0 in the field TCDF. (An "image element" is that portion of an image that has a unique time of imaging assigned to it, per the time model contained in the RSMIDA TRE for that image.) If the value of TCDF is 2, errors are modeled as "restricted image element" errors, and the above equation is only applicable when τ represents the time difference between two pixels in the same image. If the two pixels are from different images, then $\rho(\tau) = 0$ for all values of τ.

If the value of TCDF is 1, errors are modeled as "image" errors, and the above equation is applicable regardless the images associated with the two pixels; however, the definition of τ is changed from the time between two pixels, to the time between the two images that the two pixels are from. (The "image element", in this case, "becomes" the entire RSM image domain.) The time of an image is defined as the time of its center pixel within the RSM image domain. Thus, if the two images are the same image, τ has a value of zero. And, in particular, two pixels from the same image have a correlation value of 1.0. (Note that, regardless the value of the field TCDF, if the two pixels associated with the time difference τ are the same pixel, or actually within the same image element, $\tau = 0$ and $\rho(\tau) = 1.0$.)

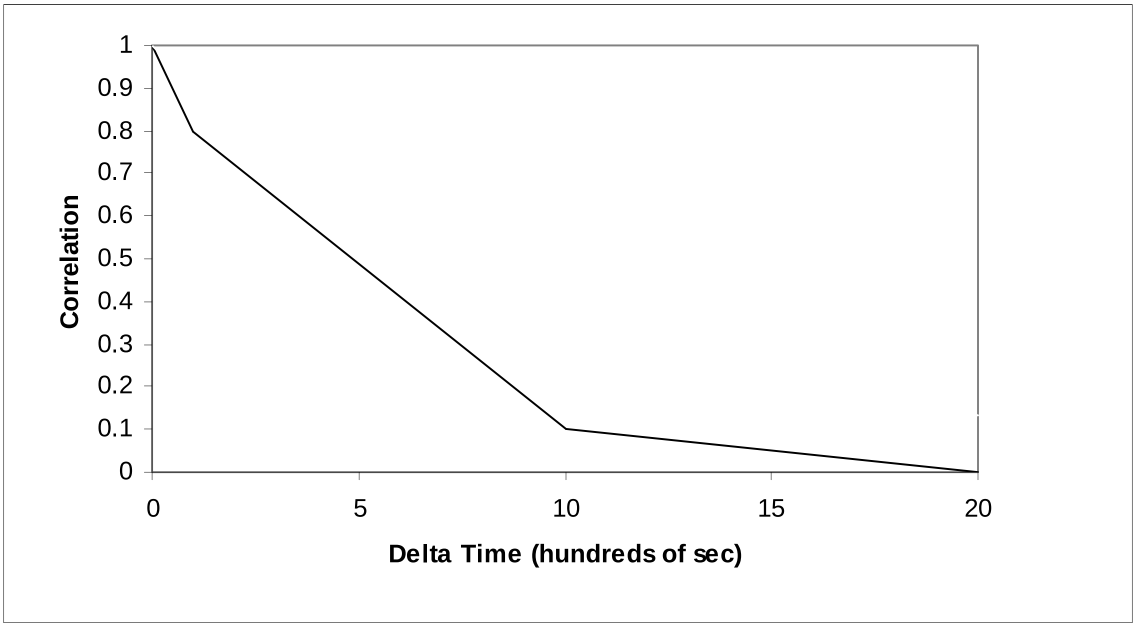

Figure 5: Example of piece-wise linear correlation function $\rho(\tau)$

3.3.1 Additional computations if non-stationary stochastic error model

As mentioned previously, when the values of C_{Si^*k} and/or $\rho_{Si^*k}(\tau)$ vary across the images i^*, original adjustable parameter errors associated with independent subgroup k are modeled as non-stationary stochastic errors. The error model is an approximation based on the combination of n wide-sense stationary error models, each associated with an image referenced by the indirect error covariance ($1 \le i^* \le n$). Each wide-sense stationary error model is based on the original sensor model error (auto) covariance and time correlation data contained in the corresponding image's RSMECX TRE.

(Note that although the values of C_{Si^*k} and/or $\rho_{Si^*k}(\tau)$ vary across the images i^* in the non-stationary case, the associated original adjustable parameters (identities) and their type of error remain invariant across the images i^* for each subgroup k. In particular, the values of fields ERRCVG, NCSEG, CORSEG, and TAUSEG may vary, but the values of fields IGN, NUMOPG, and TCDF do not.)

Computation of the RSM indirect error covariance in support of a non-stationary stochastic error model is identical to that specified previously for an assumed wide-sense stationary stochastic error model, with exceptions detailed below.

The block entry of C_{Sij} corresponding to independent subgroup k and images $i*$ and $j*$ is modified as follows: $\rho_{Si*k}(\tau)C_{Si*k} \to \overline{\rho}_{Si*j*k}(\tau)C_{Si*k}^{1/2}C_{Sj*k}^{T/2}$. The matrix superscripts correspond to a matrix square root based on a Cholesky factorization (decomposition) of the corresponding error covariance matrix, i.e., for a general positive definite error covariance C, $C = C^{1/2}C^{T/2}$, where $C^{1/2}$ is in lower triangular form and its transpose $C^{T/2}$ is in upper triangular form.

The scalar correlation function $\overline{\rho}_{Si*j*k}(\tau)$ is an "ensemble" correlation function, and consists of an average of correlation functions, each correlation function associated with an image that is correlated with both image $i*$ and image $j*$, i.e.,

$\overline{\rho}_{Si*j*k}(\tau) = (1/n_{r*})\sum_{r*}\rho_{Sr*k}(\tau)$, where each image $r*$ is correlated with both images $i*$ and $j*$. There are a total of $n_{r*} \le n$ such images. (The independent subgroup k is also applicable, but no longer mentioned for ease of description.)

In general, two images are correlated either directly or indirectly. If the (piece-wise linear decay) correlation function for image $i*$ decays to zero at the "cut-off" time τc_{i*}, and the correlation function for image $j*$ decays to zero at the "cut-off" time τc_{j*}, images $i*$ and $j*$ and directly correlated when $\tau b_{i*j*} \le \max(\tau c_{i*}, \tau c_{j*})$, where τb_{i*j*} is defined as the smallest time interval possible between an arbitrary pixel location in image $i*$ and an arbitrary pixel location in image $j*$.

Two images $i*$ and $j*$ are indirectly correlated if there is a "chain" of directly correlated images that "connects" them. For example, if image $i*$ is directly correlated with image $a*$, and image $a*$ is directly correlated with image $b*$, and image $b*$ is directly correlated with image $j*$.

The following are examples of groups of correlated images, where t_{i*} designates image $i*$'s time of first pixel (seconds), and dt_{i*} designates its image "scan" duration (seconds):

(1) images $1*$, $2*$, $3*$; $t_{1*} = 0$, $t_{2*} = 100$, $t_{3*} = 300$; $dt_{1*} = dt_{2*} = dt_{3*} = 10$; $\tau c_{1*} = \tau c_{2*} = \tau c_{3*} = 3000$; all possible image pairs are directly correlated, thus the ensemble correlation function applicable to any image pair $(i*, j*)$ within the set of three images is the same function, $\overline{\rho}_{Si*j*k}(\tau) = (1/3)[\rho_{S1*k}(\tau) + \rho_{S2*k}(\tau) + \rho_{S3*k}(\tau)]$.

(2) images $1*$, $2*$, $3*$, $4*$, $5*$; $t_{1*} = 0$, $t_{2*} = 100$, $t_{3*} = 4000$, $t_{4*} = 6000$, $t_{5*} = 8000$; $dt_{1*} = dt_{2*} = dt_{3*} = dt_{4*} = dt_{5*} = 10$; $\tau c_{1*} = \tau c_{2*} = 1000$,

$\tau c_3 = \tau c_4 = \tau c_5 = 3000$; there are two different sets of correlated images $\{1^*, 2^*\}$, via direct correlation, and $\{3^*, 4^*, 5^*\}$, via direct and indirect correlation; when image pair (i^*, j^*) is from the first set, $\bar{\rho}_{Si^*j^*k}(\tau) = (1/2)[\rho_{S1^*k}(\tau) + \rho_{S2^*k}(\tau)]$, when from the second set, $\bar{\rho}_{Si^*j^*k}(\tau) = (1/3)[\rho_{S3^*k}(\tau) + \rho_{S4^*k}(\tau) + \rho_{S5^*k}(\tau)]$, and when i^* is in one set and j^* in the other, $\bar{\rho}_{Si^*j^*k}(\tau) = 0$.

The remainder of this RSMECX TRE description is independent of whether errors are modeled as wide-sense stationary or non-stationary stochastic errors.

3.3.2 Comparison to direct error covariance

Note that an RSM indirect error covariance, assembled with TCDF=1 (“image” errors) for all original adjustable parameters and corresponding to one (arbitrary) pixel in each of n images from the same sensor, has a correspondence to the RSM direct error covariance (see RSMDCX TRE). It has the same external form as the direct error covariance - both the indirect error covariance and the direct error covariance reference n sets of RSM adjustable parameter “image” errors across the n images. In particular, assuming all n images were known prior to TRE generation, a direct error covariance could be built identical to the assembled indirect error covariance.

However, in general, the direct error covariance can be more general internally than the indirect error covariance. That is, the correlation between RSM adjustable parameters (errors) between images does not have to conform to an a priori model (piece-wise linear decay for associated original adjustable parameter errors) inherent with the indirect error covariance. The direct error covariance can also be more general externally – it can correspond to images from different sensors with different sets of active RSM adjustable parameters. On the other hand, if the number of images (n) is reasonably large and from the same sensor, the direct error covariance’s RSMDCX TRE requires more image support data bandwidth than does the indirect error covariance’s RSMECX TRE. Also, all images referenced by the direct error covariance must be specifically identified prior to its generation, and all RSM adjustable parameter errors can be modeled as “image” errors (TCDF=1) only. Neither of these restrictions apply to the indirect error covariance.

3.3.4 Indirect error covariance in “direct error covariance form”

The RSM indirect error covariance can be built in a “direct error covariance form”, directly suitable for use in a triangulation solution process, if so desired. In the “direct error covariance form”, the indirect error covariance is applicable to the specified images, but independent of image row/column (pixel) location(s). If there are n images, m_i adjustable parameter for image $i = 1,..,n$, the indirect error covariance is an $\sum_{i=1}^{n} m_i \times \sum_{i=1}^{n} m_i$ matrix. All errors in each independent subgroup are assumed “image” errors, as opposed to “image element” or “restricted image element” errors, regardless the value of the TCDF field in the RSMECX TRE. (This is an approximation when the TCDF value specifies either “image element” or “restricted image element” errors. It is not an approximation when the TCDF value specifies “image” errors, as they are 100% positively correlated across all pixel locations in the image, by definition.) In addition, if the TCDF field for a particular independent subgroup specifies “image element” errors, the corresponding time correlation function is assumed applicable to the time between images. If the TCDF field specifies “restricted image element” errors, the correlation between images is assumed zero.

The remainder of this RSMECX description assumes that the RSM indirect error covariance is not in “direct error covariance form”, i.e., it is generally applicable to multiple pixel locations per image and multiple images, as described earlier.

3.4 RSM adjustable parameter identification and definitions in support of the indirect error covariance

3.4.1 Overview

As mentioned previously, the active RSM adjustable parameters for the associated image are identified in this TRE (RSMECX). Application of corresponding C_{Rij} blocks contained in the indirect error covariance CR requires the complete definition of these active adjustable parameters. In particular, their definition is required in order to compute the partial derivatives of image measurements with respect to the adjustable parameters that are referenced by C_{Rij}, in support of error propagation. The following provides remaining details.

Active RSM adjustable parameters for the associated image are either active RSM "image-space" adjustable parameters or active RSM "ground-space" adjustable parameters, as specified by field APTYP in the RSMECX TRE.

RSM "image-space" adjustable parameters correspond to adjustable parameters that adjust an image row coordinate value (r) and an image column coordinate value (c) corresponding to an arbitrary ground point location $X = \begin{bmatrix} x & y & z \end{bmatrix}^T$. An individual adjustable parameter either adjusts an image row coordinate value or an image column coordinate value. The adjustments Δr and Δc corresponding to adjustable parameters ap_{rijk} and ap_{cijk} are computed as follows:

$$\Delta r = ap_{rijk} x^i y^j z^k$$

$$\Delta c = ap_{cijk} x^i y^j z^k$$

The adjustable parameters (ap_{rijk} and ap_{cijk}) are uniquely identified by their collective x, y, z powers (exponents) and whether they adjust image row or image column coordinates. The coordinates x, y, and z correspond to normalized ground point coordinates expressed in a Local coordinate system. Normalization is performed by an offset and scale factor for each coordinate. These normalization parameters are in contiguous fields (NSFX-NOFFZ), and allow for an approximate range of (-1,1) for each ground coordinate value. An example of their application for normalizing the y coordinate is as follows:

$$y \rightarrow (y - offset_y) / scale_y \, .$$

Because the ground coordinates are normalized, all "image-space" adjustable parameters have units of pixels, as do the corrections $\mathrm{A}r$ and Δc. Normalization of the Local coordinates helps to insure overall stability since the value of $x^i y^j z^k$ that multiplies an adjustable parameter during an image row or column adjustment can become extremely large if coordinates were not normalized for large images and large exponents.

There are two possible choices for the Local coordinate system, either Local Rectangular or Local Non-Rectangular, as specified in field LOCTYP. For Local Non-Rectangular, x, y, and z correspond to the ground point's corresponding image row coordinate, image column coordinate, and geodetic height, respectively. The Local Rectangular coordinate system is defined as a rectangular system that is offset and rotated relative to the WGS-84 coordinate system. It is typically specified as a local tangent plane system centered within the RSM image domain’s footprint at a nominal height above the ellipsoid and rotated to be aligned as follows: the z-axis is aligned with the imaging locus direction (line-of-sight vector for an electro-optical sensor), the x-axis is aligned with the image line (“sweep” or "scan") direction, and the y-axis completes a right-handed rectangular system. (When the Local Rectangular coordinate system is footprint centered, corresponding Local Rectangular coordinate normalization offsets, such as $offset_y$, typically have a value of zero.)

Figure 6 illustrates a typical Local Rectangular coordinate system. Specification of a Local Rectangular coordinate system is unique to the associated image and based on contiguous fields (XUOL-ZUZL) as detailed later in this introduction.

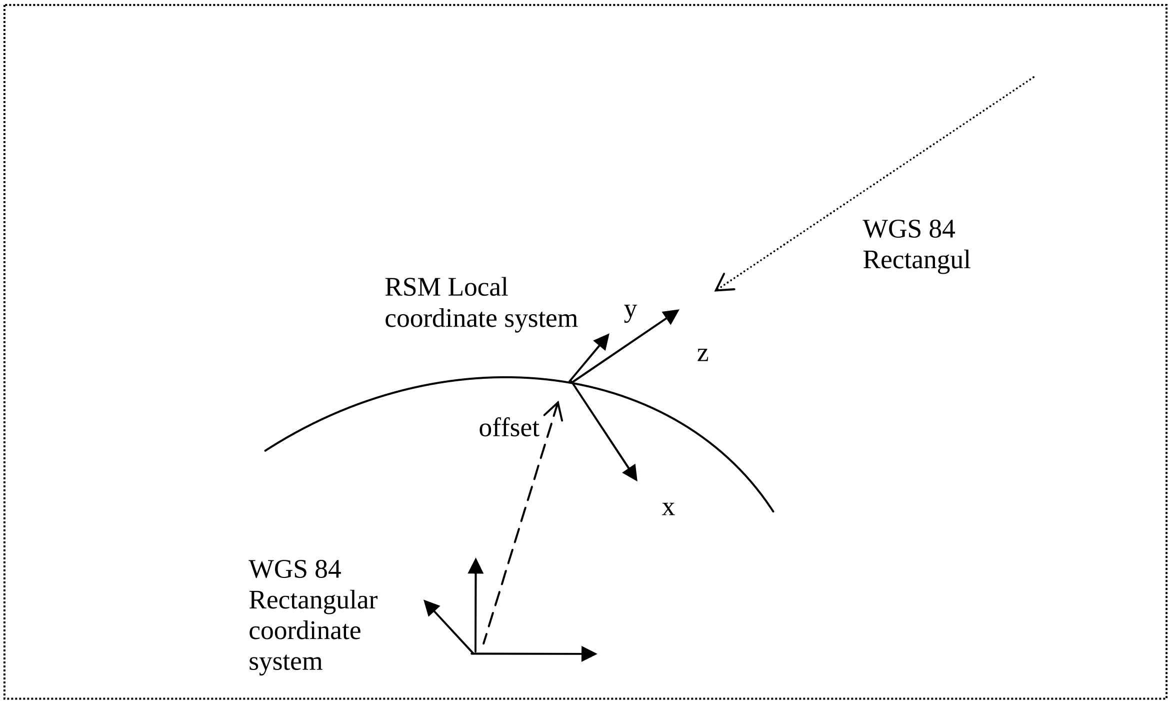

Figure 6: Example of RSM Local Rectangular Coordinate System

Note that the choice of Local Rectangular or Local Non-rectangular is provided for flexibility. The Local Rectangular coordinate system inherits general analytic advantages associated with rectangular (orthonormal) coordinates, and its absolute orientation is insensitive to any abrupt changes in imaging geometry across the imaging time interval. The Local Non-Rectangular coordinate system may provide advantages when very long images are (smoothly) scanned due to significant changes in instantaneous image geometry from one end of the image to the other. The coordinate system is continuously in alignment with these changes.

RSM "ground-space" adjustable parameters reference normalized Local Rectangular coordinates only. The coordinate system is typically specified as a local tangent plane system centered within the RSM image domain’s footprint at a nominal height above the ellipsoid (z-axis vertical). An individual "ground-space" adjustable parameter is either a parameter associated with a "seven parameter" adjustment or a "rate" adjustment. The seven parameter adjustment is defined as follows, where the symbols $\{\delta x \;\; \delta y \;\; \delta z \;\; \delta\alpha \;\; \delta\beta \;\; \delta\kappa \;\; \delta s\}$ correspond to the adjustable parameters:

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + \begin{bmatrix} \delta s & \delta\kappa & -\delta\beta \\ -\delta\kappa & \delta s & \delta\alpha \\ \delta\beta & -\delta\alpha & \delta s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The vector on the left side of the above equation corresponds to adjustments in the un-normalized coordinates of the ground point expressed in Local Rectangular coordinates with units of meters. The vector on the far right side of the equation corresponds to normalized coordinates of the ground point expressed in Local Rectangular coordinates. Because these coordinates are unit-less, the adjustable parameters all have units of meters, as do the corrections $\Delta x, \Delta y, \Delta z$.

(If Local coordinate scale factors ($scale_x, scale_y, scale_z$) are set equal in value by the TRE generation process, Local coordinate values no longer necessarily range from -1.0 to 1.0. However, the above seven parameter adjustment is now equivalent to the standard photogrammetric seven parameter (small angle) transformation. It is recommended that the scale factors be set equal to a common value in a manner that yields ranges for the three local coordinates as close as possible to interval -1.0 to 1.0.)

There are nine possible ground-space adjustable parameters corresponding to rate adjustments and denoted by the symbols $\{ap_{xx}, ap_{xy}, .., ap_{zz}\}$. They adjust the un-normalized coordinates of the ground point in Local Rectangular coordinates specifically as follows:

$\Delta x = ap_{xx} x$, $\Delta x = ap_{xy} y$, $\Delta x = ap_{xz} z$,

$\Delta y = ap_{yx} x$, $\Delta y = ap_{yy} y$, $\Delta y = ap_{yz} z$,

$\Delta z = ap_{zx} x$, $\Delta z = ap_{zy} y$, $\Delta z = ap_{zz} z$.

Again, these adjustable parameters and the corrections have units of meters.

Each of the 16 possible ground-space adjustable parameters is identified by a unique four character identifier detailed in the TREs specified format (Table 3).

Note that application of RSM adjustable parameters, whether "image-space" or "ground-space" adjustable parameters, first requires converting the corresponding ground point from representation in the RSM primary ground coordinate system to the appropriate Local system. And for the case of "ground-space" adjustable parameters, the adjusted ground point must also be converted back to the RSM primary coordinate system.

Figure 7 presents the RSM adjustable ground-to-image function $h(X, R)$, where X corresponds to the un-normalized three dimensional ground point in the RSM primary ground coordinate system. The functions $I\_adj(X, R)$ and $X\_adj(X, R)$ apply the previously documented adjustment equations for active "image-space" and "ground-space" adjustable parameters, respectively. (The functions also internally convert X from the primary system to the (normalized) Local system.)

$\Delta I = [\Delta r \quad \Delta c]^T$ denotes the summed effects at ground point location X of all active RSM "image-space" adjustable parameters. For example, if the active "image-space" adjustable parameters correspond to (combined) powers in x and y less than or equal to one: $\Delta r = ap_{r000} + ap_{r100} \cdot x + ap_{r010} \cdot y$, and $\Delta c = ap_{c000} + ap_{c100} \cdot x + ap_{c010} \cdot y$.

$\Delta X = [\Delta x \quad \Delta y \quad \Delta z]^T$ denotes the summed effects at ground point location X of all active RSM "ground-space" adjustable parameters.

The vector R represents the active RSM adjustable parameters in the order that they are specified in this populated TRE, e.g., vector element two corresponds to the second active adjustable parameter identified in the populated TRE (see Table 3). (Internally, the RSM ground-to-image function $g(X)$ is actually performed with respect to normalized coordinates. The RSM ground-to-image function handles all required normalization and un-normalization, as described in the RSMPCA TRE and the RSMGGA TRE.)

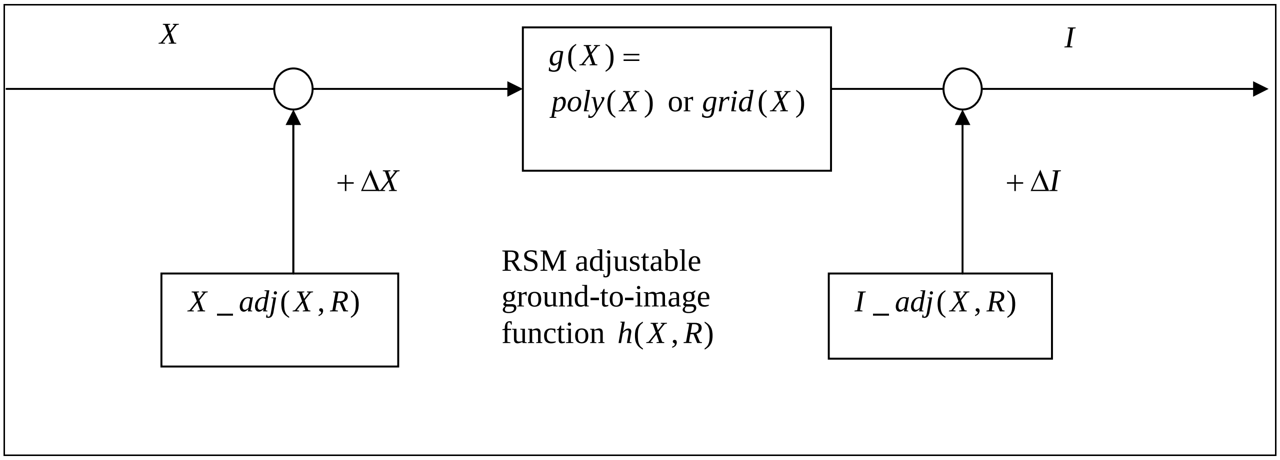

Figure 7: RSM adjustable ground-to-image function

The total number of active "image-space" adjustable parameters is specified in field (NISAP). Individual active adjustable parameters are identified in contiguous fields (XPWRR-ZPWRR) and contiguous fields (XPWRC-ZPWRC). The index of an active adjustable parameter into a cross-covariance block for the associated image (or, equivalently, the row of the corresponding mapping matrix) corresponds to the order it is identified in the populated TRE.

The total number of active "ground-space" adjustable parameters is specified in field (NGSAP). Individual active parameters are identified in field (GSAPID). The index of an active parameter into a cross-covariance block for the associated image (or, equivalently, the row of the corresponding mapping matrix) corresponds to the order it is identified in the populated TRE

As mentioned earlier, active RSM adjustable parameters require definition and identification in order to support error propagation. In particular, to project the (RSM support data) image error covariance (more generally, C_{Rij}) to image space via the partial derivatives of image coordinates with respect to the active adjustable parameters. These derivatives are computed for the various active adjustable parameters by taking the appropriate derivatives of the previous equations.

For example, the partial derivative of the image column coordinate with respect to an "image-space" adjustable parameter is computed as follows:

$$\partial c / \partial ap_{cijk} = \partial \Delta c / \partial ap_{cijk} = \partial (ap_{cijk} x^i y^j z^k) / \partial ap_{cijk} = x^i y^j z^k$$

(Recall that in the above the ground point's location (x, y, z) is represented as normalized coordinates in the Local system.)

Partial derivatives of image coordinates with respect to "ground-space" adjustable parameters are more involved because they adjust image coordinates indirectly. For example, the partial derivative of the image row coordinate with respect to the "ground-space" adjustable parameter ap_{xy} is computed as follows:

$$\partial r / \partial ap_{xy} = (\partial r / \partial X)(\partial X / \partial ap_{xy})$$

In this equation, X represents the ground point's location in the primary coordinate system, and $\partial r / \partial X$ (1×3) is readily computed from the RSM ground-to-image function for the row coordinate, i.e., $\partial r / \partial X = \partial g_r(X) / \partial X$.

The (3×1) $\partial X / \partial ap_{xy} = (\partial X / \partial X_L)(\partial X_L / \partial ap_{xy})$, where X_L represents the ground point location in the (un-normalized) Local Rectangular coordinate system. $\partial X / \partial X_L$ $(3x3)$ is readily computed from the Local Rectangular coordinate system and the primary coordinate system defining parameters. Also, from the definition of ap_{xy}, the (3×1) $\partial X_L / \partial ap_{xy} = [scale_x \cdot y \quad 0 \quad 0]^T$, where y is the second component of the ground point's location in normalized Local Rectangular coordinates.

3.4.2 Basis Option

When this option is invoked (APBASE=Y), the set of RSM adjustable parameters specified in this TRE (as described previously) become a "basis" set of adjustable parameters. Symbolically, they are contained in the vector R, assumed to have n elements. Another set of RSM adjustable parameters is defined as a linear combination of the elements of R. Symbolically, this new set is contained in the vector R', where $R' = AR$, and the matrix A is $m \times n$, $m \leq n$, with the rank of A equal to m. The vector R' contains the (new) set of active adjustable parameters.

The RSM image error covariance (more generally, C_{Rij}) assembled from data in this TRE is now with respect to R'. Equivalently, the mapping matrix contained in this TRE is also now with respect to R'. The field NPAR now corresponds to the number of elements (m) in R'. The field NBASIS corresponds to the number of elements (n) in R.

Typically, the mapping matrix and A are determined during generation of this TRE from an initial error covariance with respect to the basis set R using principal components analysis (see RSMDCX for an overview of the process and its potential benefits). R is also a linear combination of R' based on the pseudo-inverse of A, designated as $A^{\#}$ (or AI). Thus, $R = A^{\#}R'$, where $A^{\#} = A^T(AA^T)^{-1} = A^T$.

When this option is on, the RSMECX TRE contains the identification of the adjustable parameters that make up the elements of the basis (R), the matrix A that maps the basis (R) to the set of active adjustable parameters (R'), and the mapping matrix (Φ) with respect to the set of active RSM adjustable parameters R'. Corresponding exploiter functionality is invoked by specification of the appropriate partial derivatives of image coordinates with respect to R', and the update of the RSM ground-to-image function from values of R' solved for during an RSM adjustment (triangulation):

(1) $\partial I / \partial R' = (\partial I / \partial R)A^T$, and

(2) values of R' map to values of R via $R = A^T R'$ where the subsequent R affects the ground-to-image function.

The first of these equations supports the statistical propagation of RSM support data errors to image space, i.e., the projection of the associated image's error covariance (more generally, C_{Rij}) assembled from data in this TRE to image space. (The computation of the partial derivatives $\partial I / \partial R$ was discussed previously). The second equation maps adjustments contained in R' to R for subsequent application in the RSM adjustable ground-to-image function for the associated image.

3.4.3 Local Rectangular coordinate system details

The following defines the Local Rectangular coordinate system relative to the WGS 84 Rectangular coordinate system. The contiguous fields XUOL through ZUOL specify the origin (offset) of the Local coordinate system relative to the WGS 84 Rectangular coordinate system, and the contiguous fields XUXL through through ZUZL elements of the rotation matrix. These fields are provided in this TRE.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{LOCAL}} = \begin{bmatrix} XUXL & YUXL & ZUXL \\ XUYL & YUYL & ZUYL \\ XUZL & YUZL & ZUZL \end{bmatrix} \cdot \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{WGS}-84} - \begin{bmatrix} XUOL \\ YUOL \\ ZUOL \end{bmatrix} \right).$$

Note that the definition of the Local Rectangular coordinate system is also redundantly supplied in other TREs for the associated image. Also, in order to convert a ground point X represented in the RSM primary ground coordinate system (e.g., Geodetic) to the Local Rectangular coordinate system, it must first be converted from the RSM primary system to the WGS 84 Rectangular coordinate system.

3.5 Unmodeled error covariance

The RSMECX TRE may also contain information specifying an unmodeled error covariance corresponding to multiple pixel locations within the associated image's RSM image domain. Unmodeled errors represent the summed effects of all errors that can not be represented as RSM adjustable parameter errors. If present, they are typically relatively non-systematic, "high frequency" errors. Representation of unmodeled errors is done directly in image space. The corresponding unmodeled error covariance is applicable to errors at an arbitrary time (pixel location) in the associated image's RSM image domain. These unmodeled errors are also assumed correlated between pixel locations, as represented by a correlation model as a function of number of rows between pixel locations and a correlation model as a function of number of columns between pixel locations. The unmodeled errors are assumed uncorrelated between images. Specifically, the unmodeled error covariance (CU) for the associated image is defined as follows:

$$CU = \begin{bmatrix} C_{U11} & C_{U12} & . & . & C_{U1q} \\ . & C_{U22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & C_{Uqq} \end{bmatrix},$$

where there are an assumed q pixel locations of interest within the RSM image domain, and C_{Uij} is the 2×2 error cross-covariance between pixels i and j given by:

$$C_{Uij} = \rho_U(\Delta u_{ij})\rho_V(\Delta v_{ij})C_U .$$

The upper triangular portion of the symmetric 2×2 error covariance C_U is specified in the fields URR, URC, and UCC. The correlation function $\rho_U(\Delta u_{ij})$ is specified in the fields UNCSR, UCORSR, and UTAUSR. The correlation function $\rho_V(\Delta v_{ij})$ is specified in the fields UNCSC, UCORSC, and UTAUSC. The row and column distances between pixels i and j are defined as Δu_{ij} and Δv_{ij}, respectively. Both the scalar correlation functions $\rho_U(\Delta u_{ij})$ and $\rho_V(\Delta v_{ij})$ are of the same form as the correlation function used for the indirect error covariance. However, τ is redefined from being the time between pixels to the distance (number of rows or columns) between pixels. Also, there is no explicit counterpart to the field TCDF for these functions. Unmodeled errors are assumed “restricted image element errors”, where the image element is the pixel.

Note that unmodeled errors are typically not applicable. Also, if unmodeled errors are applicable, modeled errors can be represented by either the indirect error covariance (RSMECX) or the direct error covariance (RSMDCX). For both of these reasons, the unmodeled error covariance information is conditional within this (RSMECX) TRE, as defined by fields INCLIC and INCLUC. If the RSM support data for the associated image contains both the RSMECX TRE and the RSMDCX TRE, the latter takes precedence for modeled errors, and the former specifies unmodeled error if INCLUC=Y.

3.6 Matrix element ordering

Finally, regarding the ordering of matrix elements in this TRE, the error covariance associated with the original sensor model adjustable parameters for a particular independent subgroup is in an upper triangular form with matrix elements in field ERRCVG. The upper triangular matrix is provided in row major order (the top row first, followed by the second row less the leftmost column, all the way to the rightmost element of the bottom row). The associated image's mapping matrix is a full matrix in row major order with elements in field MAP. If the basis option is on, the A matrix is a full matrix in row major order with elements in field AEL.

The mapping matrix is in row major order with one row per RSM adjustable parameter and one column per original adjustable parameter. Note that it is not in upper triangular form.

3.7 RSMECX format

Table 3 specifies the detailed format for the Replacement Sensor Model Error Covariance (RSMECX) TRE.

RSMECX – Replacement Sensor Model Error Covariance

Field	Name/Description	Size	Format	Units	Estimated Value Range	Type
TAG Information						
CETAG	Unique Extension Type Identifier Unique TRE identifier.	6	BCS-A	N/A	RSMECX	R
CEL	Length of User-Defined Data Length in bytes of data contained in subsequent fields. (TREs length is 11 plus the value given in the CEL field)	5	BCS-N	bytes	00369 to 69570	R
Image Information						
IID	Image Identifier. This field contains a character string that uniquely identifies the original full image that corresponds to the associated image. This is not to be confused with the identification of an image derived by filtering, chipping, re-sampling, or other such image to image transformations. The image identifier is left justified with trailing spaces.	80	BCS-A	N/A	N/A All spaces if unavailable	<R>
EDITION	RSM Image Support Data Edition. This field contains a character string that uniquely identifies the RSM support data for the associated original full image. It is to consist of an identifier of up to 20 characters for the processor that generated the RSM support data, to which is appended up to 20 characters that are unique to that processor.	40	BCS-A	N/A	N/A	R
TID	Triangulation ID. This field contains an identifier that is unique to the most recent process after RSM support data generation that led to the adjustments and/or error covariance in this RSM support data edition.	40	BCS-A	N/A	N/A	R
TRE Covariance Options						
INCLIC	Include Indirect Error Covariance Flag. If the value of this field is Y, the indirect error covariance information is included in this TRE.	1	BCS-A	N/A	Y or N	R
INCLUC	Include Unmodeled Error Covariance Flag. If the value of this field is Y, the unmodeled error covariance information is included in this TRE.	1	BCS-A	N/A	Y or N	R
…if (INCLIC = Y) then include the following fields:						
NPARO	Number of Original Adjustable Parameters. This field contains the number of original adjustable parameters of the associated image. It is both the row and column dimensions of the (unmapped) original image error covariance and the original image error cross-covariance. The maximum allowed number of original adjustable parameters is 36.	2	BCS-N	N/A	01 to 36	C
IGN	Number of Independent Subgroups. This field contains the number of independent adjustable parameter (error) subgroups associated with the original adjustable parameters of the associated image.	2	BCS-N	N/A	01 to 36	C

CVDATE	Version Date of the Original Image Error Covariance. Date representing the version of the error model applicable to the original image error covariance. If populated, and two images are from the same sequence of images from the same sensor, and if the values of CVDATE are different in the two RSMECX TREs, all original adjustable parameter (errors) are assumed uncorrelated between the images.	8	BCS-A	N/A	YYYYMMDD (YYYY=four digit year, MM=two digit month; DD=two digit day) Population optional Default is all spaces Value must correspond to a valid date	<C>
RSM Adjustable Parameter Identification for the associated image						
NPAR	Number of Active RSM Adjustable Parameters. This field contains the total number of active RSM adjustable parameters for the associated image. The value of this field is the row dimension of any RSM cross-covariance (block) for the associated image, as well as the row dimension of the associated image's mapping matrix (field MAP). It is also both the row and the column dimension of any RSM (auto) covariance (block) for the associated image. NPAR's maximum value of 36 constrains an RSM covariance block to be reasonable size. (If the "basis" option is off (APBASE=N), NPAR=NISAP if APTYP=I, and NPAR=NGSAP if APTYP=G. If the basis option is on (APBASE=Y), NPAR corresponds to the number of (new) active adjustable parameters and the number of row in the matrix A, as described for field APBASE.)	2	BCS-N	N/A	01 to 36	R
APTYP	Adjustable Parameter Type. This field identifies whether RSM adjustable parameters are "image-space" (APTYP=I) or "ground-space" (APTYP=G) adjustable parameters.	1	BCS-A	N/A	I or G	C
LOCTYP	Local Coordinate System Identifier. The field identifies whether the Local coordinate system references rectangular ground coordinates (LOCTYP=R) or non-rectangular (image row/image column/geodetic height) coordinates (LOCTYP=N). If RSM adjustable parameters are specified as "ground-space" (APTYP=G), the only valid value is LOCTYP=R.	1	BCS-A	N/A	R or N	C
Normalization Factors for the Local System						
NSFX	Normalization Scale Factor for X. This field contains the normalization scale factor for the x component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NSFY	Normalization Scale Factor for Y. This field contains the normalization scale factor for the y component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C

NSFZ	Normalization Scale Factor for Z. This field contains the normalization scale factor for the z component of the Local coordinate system	21	BCS-A	meters	±9.99999999999999E±99	C
NOFFX	Normalization Offset for X. This field contains the normalization offset for the x component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NOFFY	Normalization Offset for Y. This field contains the normalization offset for the y component of the Local coordinate system. Units are meters if field LOCTYP=R or pixels if LOCTYP=N.	21	BCS-A	meters or pixels	±9.99999999999999E±99	C
NOFFZ	Normalization Offset for Z. This field contains the normalization offset for the z component of the Local coordinate system.	21	BCS-A	meters	±9.99999999999999E±99	C
Local Rectangular Ground Coordinates Detailed Definition for Associated image						
...if (LOCTYP=R)						
XUOL	Local Coordinate Origin (XUOL). This field provides the WGS 84 *X* coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E+99	C
YUOL	Local Coordinate Origin (YUOL). This field provides the WGS 84 *Y* coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E+99	C
ZUOL	Local Coordinate Origin (ZUOL). This field provides the WGS 84 *Z* coordinate of the origin of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	meters	±9.99999999999999E+99	C
XUXL	Local Coordinate Unit Vector (XUXL). This field provides the WGS 84 *X* component of the unit vector defining the *X*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
XUYL	Local Coordinate Unit Vector (XUYL). This field provides the WGS 84 *X* component of the unit vector defining the *Y*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
XUZL	Local Coordinate Unit Vector (XUZL). This field provides the WGS 84 *X* component of the unit vector defining the *Z*-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C

YUXL	Local Coordinate Unit Vector (YUXL). This field provides the WGS 84 Y component of the unit vector defining the X-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
YUYL	Local Coordinate Unit Vector (YUYL). This field provides the WGS 84 Y component of the unit vector defining the Y-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
YUZL	Local Coordinate Unit Vector (YUZL). This field provides the WGS 84 Y component of the unit vector defining the Z-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
ZUXL	Local Coordinate Unit Vector (ZUXL). This field provides the WGS 84 Z component of the unit vector defining the X-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
ZUYL	Local Coordinate Unit Vector (ZUYL). This field provides the WGS 84 Z component of the unit vector defining the Y-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
ZUZL	Local Coordinate Unit Vector (ZUZL). This field provides the WGS 84 Z component of the unit vector defining the Z-axis of the Local (rectangular) coordinate system. This coordinate system is part of the RSM adjustable parameters definition for the image.	21	BCS-A	N/A	±9.99999999999999E+99 Value consistent with fields XUXL through ZUZL forming an orthogonal matrix	C
...end if (LOCTYP=R)						
RSM Adjustable Parameter Basis Option						

APBASE	Basis Option. This field indicates whether the RSM adjustable parameters "basis" option is on (APBASE=Y). If this option is off (APBASE=N), the RSM adjustable parameters specified in the following fields are the active RSM adjustable parameters. The order (component number) of an active RSM adjustable parameter is the order in which it is specified. If this option is on, the RSM adjustable parameters specified in the following fields are the basis set of RSM adjustable parameters. The order (component number) of a basis RSM adjustable parameter is the order in which it is specified. If this option is on, the active RSM adjustable parameters are a linear combination of the basis set of RSM adjustable parameters. The matrix A (field AEL) maps the basis set to the active set of RSM adjustable parameters. In addition, the pseudo-inverse of the matrix A is equal to the matrix A transpose. It maps the active set to the basis set of RSM adjustable parameters. The matrix A is mxn, where $m \le n$ and the rank of A equals m. The number of adjustable parameters (n) in the basis set is specified in field NBASIS. The number of active adjustable parameters (m) is specified in the field NPAR. The RSM image error covariance is always with respect to the active RSM adjustable parameters. For example, the second active RSM adjustable parameter corresponds to row 2 and column 2 of the image (auto) covariance, and corresponds to row 2 and column k of the cross-covariance of the associated image with image k.	1	BCS-A	NA	Y or N	C
“Image-space” Adjustable Parameters						
...if (APTYP=I)						

NISAP	Number of Image-Space Adjustable Parameters. This field contains the total number of image-space adjustable parameters. If the basis option is off (APBASE=N), specified image-space adjustable parameters are the active RSM adjustable parameters. The total number of image-space adjustable parameters is constrained as follows: (0<NPAR=NISAP=(NISAPR + NISAPC)<37). NISAPR is the number of image-space adjustable parameters that affect the image row-coordinate, and NISAPC the number that affect the image column-coordinate. If the basis option is on (APBASE=Y), specified image-space adjustable parameters are the basis RSM adjustable parameters. The total number of image-space adjustable parameters making up the basis set is constrained as follows: (0<NBASIS=NISAP=(NISAPR + NISAPC)<100).	2	BCS-A	N/A	1-36 (if APBASE=N) 1-99 (if APBASE=Y)	C
NISAPR	Number of Image-Space Adjustable Parameters for Image Row Coordinate. This field provides the total number of image-space adjustable parameters that adjust the image row coordinate. The general form for the row coordinate adjustment (Δr) corresponding to an adjustable parameter (ap) is as follows: $\Delta r = ap_{rijk} \cdot x^i \cdot y^j \cdot z^k$, where i,j,k are the corresponding powers of normalized Local coordinates x,y,z, respectively. Each adjustable parameter has units of pixels.	2	BCS-A	N/A	0-36 (if APBASE=N) 0-99 (if APBASE=Y)	C
...Begin for each image-space adjustable parameter for row adjustment (NISAPR entries)						
XPWRR	Row Parameter Power of X. The power (exponent) of x associated with this image-space adjustable parameter for image row adjustment. This power along with the following two powers (fields) uniquely specify the adjustable parameter. x is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image row coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C

YPWRR	Row Parameter Power of Y. The power (exponent) of y associated with this image-space adjustable parameter for image row adjustment. y is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image column coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
ZPWRR	Row Parameter Power of Z. The power (exponent) of z associated with this image-space adjustable parameter for image row adjustment. z is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized geodetic height coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
...End for each image-space adjustable parameter for row adjustment						
NISAPC	Number of Image-Space Adjustable Parameters for Image Column Coordinate. This field provides the total number of image-space adjustable parameters that adjust the image column coordinate. The general form for the column coordinate adjustment (Δc) corresponding to an adjustable parameter (ap) is as follows: $\Delta c = ap_{cijk} \cdot x^i \cdot y^j \cdot z^k$, where i,j,k are the corresponding powers of normalized Local coordinates x,y,z, respectively. Each adjustable parameter has units of pixels.	2	BCS-A	N/A	0-36 (if APBASE=N) 0-99 (if APBASE=Y)	C
...Begin for each image-space adjustable parameter for column adjustment (NISAPC entries)						
XPWRC	Column Parameter Power of X. The power (exponent) of x associated with this image-space adjustable parameter for image column adjustment. This power along with the following two powers (fields) uniquely specify the adjustable parameter. x is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image row coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
YPWRC	Column Parameter Power of Y. The power (exponent) of y associated with this image-space adjustable parameter for image column adjustment. y is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized image column coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C
ZPWRC	Column Parameter Power of Z. The power (exponent) of z associated with this image-space adjustable parameter for image column adjustment. z is a normalized Local rectangular ground coordinate (LOCTYP=R) or a normalized geodetic height coordinate (LOCTYP=N)	1	BCS-A	N/A	0-5	C

...End for each image-space adjustable parameter for column adjustment
...end if (APTYP=I)
"Ground-Space" Adjustable Parameters
...if(APTYP=G)

NGSAP	Number of Ground-Space Adjustable Parameters This field provides the total number of ground-space adjustable parameters. Each ground-space adjustable parameter is either associated with a "seven parameter" adjustment or is a first order "rate" term. The general form for the seven parameter adjustment is: $\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + \begin{bmatrix} \delta s & \delta\kappa & -\delta\beta \\ -\delta\kappa & \delta s & \delta\alpha \\ \delta\beta & -\delta\alpha & \delta s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix},$ where the vector on the left side of the equation is the ground-space adjustment in Local rectangular ground coordinates (meters), the vector on the far right side of the equation is the ground point location in normalized Local rectangular ground coordinates. The seven parameters $\delta x, \delta y, \delta z, \delta\alpha, \delta\beta, \delta\kappa, \delta s$, are termed x-offset, y-offset, z-offset, rotation angle alpha, rotation angle beta, rotation angle kappa, and scale. For identification purposes in the field below, these seven parameters are assigned 4 character identifications "OFFX", "OFFY" ,"OFFZ", "ROTX", "ROTY", "ROTZ", "SCAL", respectively. Each has units of meters. There a total of 9 possible rate terms $ap_{xx}, ap_{xy}, .., ap_{zz}$, termed "XRTX", "XRTY", "XRTZ", "YRTX", "YRTY", "YRTZ", "ZRTZ", "ZRTY, "ZRTZ, respectively. Their effect is illustrated as follows for the adjustable parameter "XRTY" (ap_{xy}) and corresponding adjustment Δx : $\Delta x = ap_{xy} y$. If the basis option is off (APBASE=N), specified ground-space adjustable parameters are the active RSM adjustable parameters. If the basis option is on (APBASE=Y), specified ground-space adjustable parameters are the basis RSM adjustable parameters. The total number of ground-space adjustable parameters (NGSAP) is constrained to be between 1 and 16 regardless the value of APBASE, i.e., regardless if the basis option is on or off. If the basis option if off, NPAR=NGSAP. If the basis option if on, NBASE=NGSAP.	2	BCS-A	N/A	1-16	C

...Begin for each ground-space adjustable parameter (NGSAP entries)						
GSAPID	Ground-space Adjustable Parameter ID. This field identifies a ground-space adjustable parameter.	4	BCS-A	N/A	OFFX,OFFY,OFFZ, ROTX,ROTY,ROTZ, SCAL, XRTX, XRTY,XRTZ, YRTZ,YRTY,YRTZ, ZRTX,ZRTY,ZRTZ	C
...End for each ground-space adjustable parameter						
...end if (APTYP=G)						
...if (APBASE=Y)						
NBASIS	Number of Basis Adjustable Parameters. This field contains the number of RSM adjustable parameters in the basis set. It is equal to the number of columns in the matrix A. NBASIS=NISAP or NGSAP, depending on whether the previously identified active adjustable parameters were "image-space" or "ground-space" active adjustable parameters. The number of columns must be no less than the number of rows in the matrix A, i.e., $\text{NBASIS} \geq \text{NPAR}$. The size of the matrix A is also constrained such that $\text{NPAR*NBASIS} \leq 1296$.	2	BCS-N	N/A	1-99	C
...Begin for each A element (NPAR*NBASIS entries)						
AEL	Matrix A Element. This field contains an element of the matrix A. The elements are stored in row major order.	21	BCS-A	N/A	±9.99999999999999E±99	C
...End loop over elements of matrix A						
...end if (APBASE=Y)						
Error Covariance Data						
…Begin for each original Adjustable Parameter independent error subgroup (IGN entries)						
NUMOPG	Number of Original Adjustable Parameters in Subgroup. This field contains the number of contiguous original adjustable parameters in this independent error subgroup. (Independent error subgroups are contiguous as well.)	2	BCS-N	N/A	01 to 36 Sum of IGN entries of NUMOPG must equal NPARO	C
…Begin for each element of the original image error covariance for this independent error subgroup (1/2(NUMOPG+1)(NUMOPG) entries)						
ERRCVG	Original Error Covariance Element. This field contains an original adjustable parameter error covariance element corresponding to the independent error subgroup. The elements correspond to the upper triangular portion of the error covariance. They are in row major order.	21	BCS-A	N/A	±9.99999999999999E+99 Collectively, the ERRCVG values for the subgroup must correspond to a positive definite error covariance matrix	C
…End for each element of the original image error covariance for the independent error subgroup						

TCDF	Time Correlation Domain Flag. This field defines the type of original adjustable parameter error, and hence, the corresponding correlation function domain, for this independent error subgroup. If this field is 0, the time correlation applies to all time intervals, both within and between images. The associated errors in the original adjustable parameters are “image element errors”. If this field is 1, the time correlation applies to time intervals between images only. Time correlation for time intervals within an image is defined 100% positively correlated. The associated errors in the original adjustable parameters are “image errors”. If this field is 2, the time correlation applies to time intervals within an image only. Time correlation for time intervals between images is defined as zero. The associated errors in the original adjustable parameters are “restricted image element errors”.	1	BCS-N	N/A	0, 1, 2	C
NCSEG	Number of Correlation Segments. This field contains the number of piece-wise linear correlation segments that make up the correlation function for this independent error subgroup.	1	BCS-N	N/A	2 through 9	C
…Begin for each correlation segment (NCSEG entries)						
CORSEG	Segment Correlation Value. This field contains the correlation value applicable at the beginning of the segment. Note that the value is defined as one for the first segment (correlation segment=1), and defined as zero for the last segment (correlation segment=NCSEG). It is a nonnegative number for all segments, decreasing in value from one segment to the next.	21	BCS-A	N/A	±9.99999999999999E+99 Greater than or equal to zero and less than or equal to one Value consistent with a non-negative, convex, piece-wise linear correlation function defined by NCSEG entries of CORSEG and TAUSEG	C
TAUSEG	Segment Tau Value. This field contains the correlation time (tau) applicable at the beginning of the segment. Note that the value is defined as zero for the first segment (correlation segment=1). It is a positive number for all other segments, increasing in value from one segment to the next. Note that the values of the fields CORSEG and TAUSEG for all the segments are further constrained such that the corresponding piece-wise linear correlation function is convex (non-positive and increasing slope from one segment to the next). Also, the last segment is defined equal to zero for all tau greater than the last segment’s TAUSEG value.	21	BCS-A	seconds	±9.99999999999999E+99 Non-negative value Value consistent with a non-negative, convex, piece-wise linear correlation function defined by NCSEG entries of CORSEG and TAUSEG	C
…End for each correlation segment						
…End for each independent error subgroup						
…Loop over mapping matrix elements ((NPAR)(NPARO) entries)						
MAP	Mapping Matrix Element. This field contains the value of the next mapping matrix element, stored in row major order. The mapping matrix is used to map the associated image’s original error covariance to RSM error covariance. The mapping matrix has NPAR rows and NPARO columns.	21	BCS-A	N/A	±9.99999999999999E+99	C

…End loop over mapping matrix elements						
…End if (INCLIC = Y)						
…if (INCLUC = Y) then include the following fields:						
Unmodeled Error Covariance data						
URR	Unmodeled Row Variance. This field provides the variance of unmodeled error represented as an image row error.	21	BCS-A	pixels^2	±9.99999999999999E+99 Non-negative value	C
URC	Unmodeled Row-Column Covariance. This field provides the covariance between the unmodeled error represented as an image row error and unmodeled error represented as an image column error.	21	BCS-A	pixels^2	±9.99999999999999E+99 Collectively, URR, URC, and UCC values must correspond to a positive semi-definite (2x2) error covariance matrix	C
UCC	Unmodeled Column Variance. This field provides the variance of unmodeled error represented as an image column error.	21	BCS-A	pixels^2	±9.99999999999999E+99 Non-negative value	C
UNCSR	Number of Correlation Segments for independent variable ROW distance. This field contains the number of piece-wise linear correlation segments that make up the correlation function for unmodeled error with independent variable image row distance.	1	BCS-N	N/A	2 through 9	C
…Begin for each correlation segment (UNCSR entries)						
UCORSR	Segment Correlation Value. This field contains the correlation value applicable at the beginning of the segment. Note that the value is defined as one for the first segment (correlation segment=1), and defined as zero for the last segment (correlation segment=UNCSR). It is a nonnegative number for all segments, decreasing in value from one segment to the next.	21	BCS-A	N/A	±9.99999999999999E+99 Greater than or equal to zero and less than or equal to one Value consistent with a non-negative, convex, piece-wise linear correlation function defined by UNCSR entries of field UCORSR and field UTAUSR	C
UTAUSR	Segment Tau Value. This field contains the correlation row distance (tau) applicable at the beginning of the segment. Note that the value is defined as zero for the first segment (correlation segment=1). It is a positive number for all other segments, increasing in value from one segment to the next. Note that the values of the fields UCORSR and UTAUSR for all the segments are further constrained such that the corresponding piece-wise linear correlation function is convex (non-negative and increasing slope from one segment to the next). Also, the last segment is defined equal to zero for all tau greater than the last segment's UTAUSR value.	21	BCS-A	pixels	±9.99999999999999E+99 Non-negative value Value consistent with a non-negative, convex, piece-wise linear correlation function defined by UNCSR entries of field UCORSR and field UTAUSR	C
…End for each correlation segment						
UNCSC	Number of Correlation Segments for independent variable Column distance. This field contains the number of piece-wise linear correlation segments that make up the correlation function for unmodeled error with independent variable image column distance.	1	BCS-N	N/A	2 through 9	C
…Begin for each correlation segment (UNCSC entries)						

UCORSC	Segment Correlation Value. This field contains the correlation value applicable at the beginning of the segment. Note that the value is defined as one for the first segment (correlation segment=1), and defined as zero for the last segment (correlation segment=UNCSC). It is a nonnegative number for all segments, decreasing in value from one segment to the next.	21	BCS-A	N/A	±9.99999999999999E+99 Greater than or equal to zero and less than or equal to one Value consistent with a non-negative, convex, piece-wise linear correlation function defined by UNCSC entries of field UCORSC and field UTAUSC	C
UTAUSC	Segment Tau Value. This field contains the correlation column distance (tau) applicable at the beginning of the segment. Note that the value is defined as zero for the first segment (correlation segment=1). It is a positive number for all other segments, increasing in value from one segment to the next. Note that the values of the fields UCORSC and UTAUSC for all the segments are further constrained such that the corresponding piece-wise linear correlation function is convex (non-negative and increasing slope from one segment to the next). Also, the last segment correlation is defined equal to zero for all tau greater than the last segment's UTAUSC value.	21	BCS-A	pixels	±9.99999999999999E+99 Non-negative value Value consistent with a non-negative, convex, piece-wise linear correlation function defined by UNCSC entries of field UCORSC and field UTAUSC	C
…End for each correlation segment						
…End if (INCLUC = Y)						

Table 3: RSMECX TRE format table